

Axial Turbines

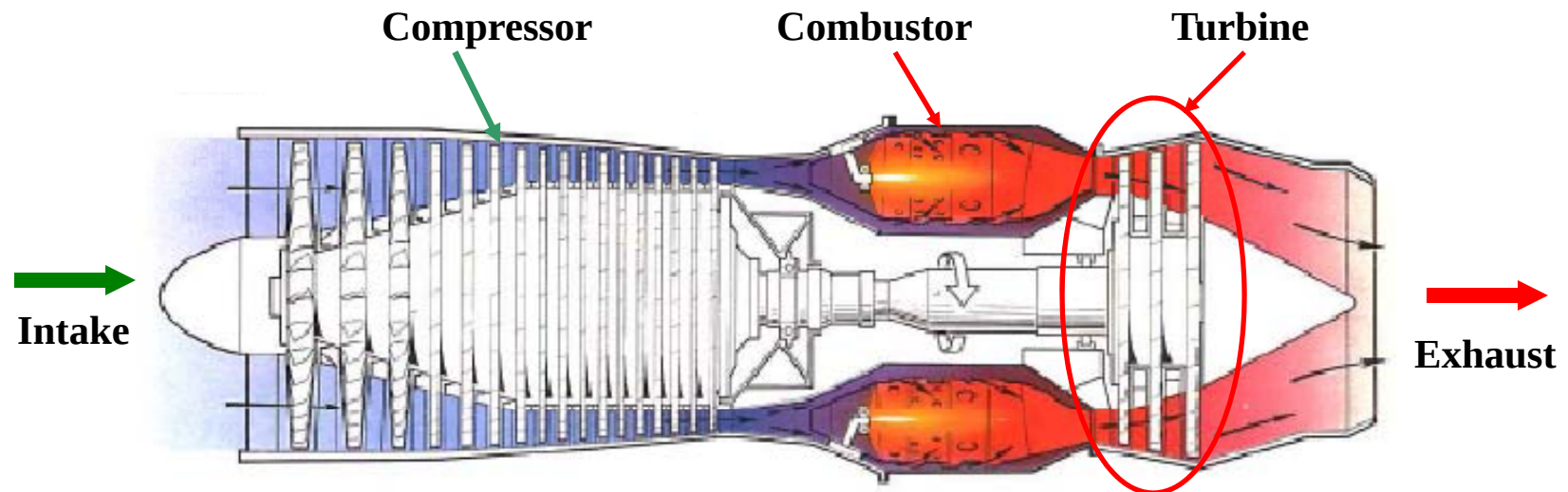
Session delivered by:

Prof. Q.H. Nagpurwala

Session Objectives

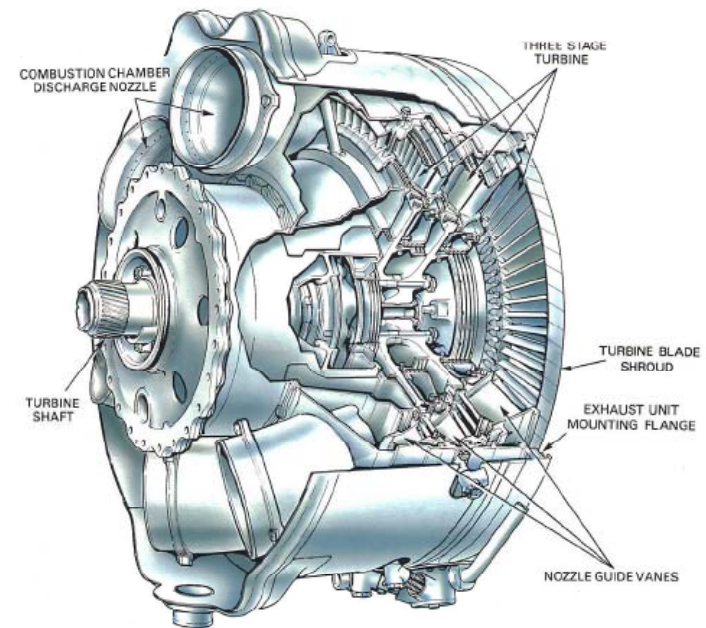
This session is intended to introduce the following:

- Construction and types of axial turbines
- Euler turbine equation and velocity triangles
- Vortex theory and radial equilibrium
- Turbine blade loading
- Loss mechanisms and loss correlations
- Turbine performance characteristics



Axial Turbine

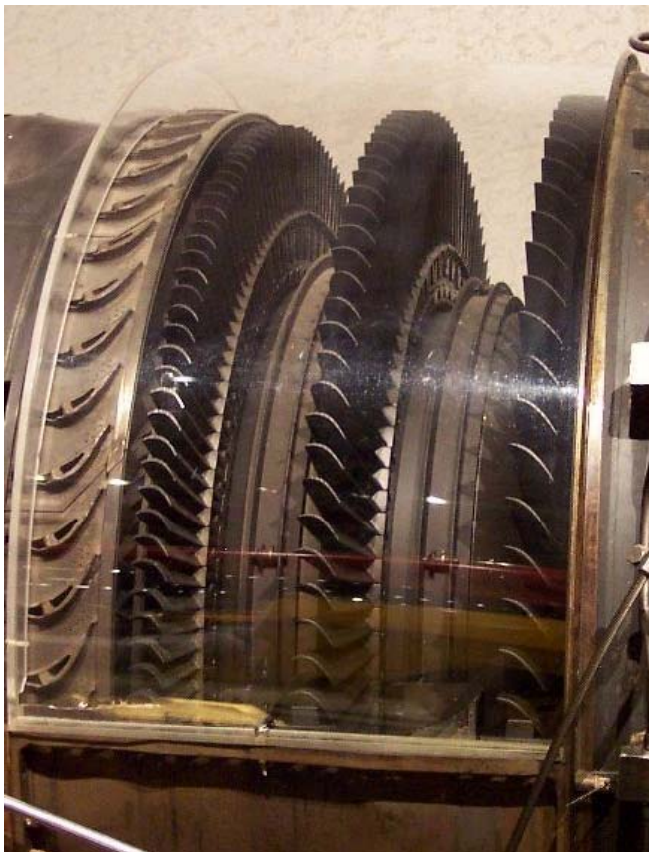
A triple-stage turbine with
single shaft system



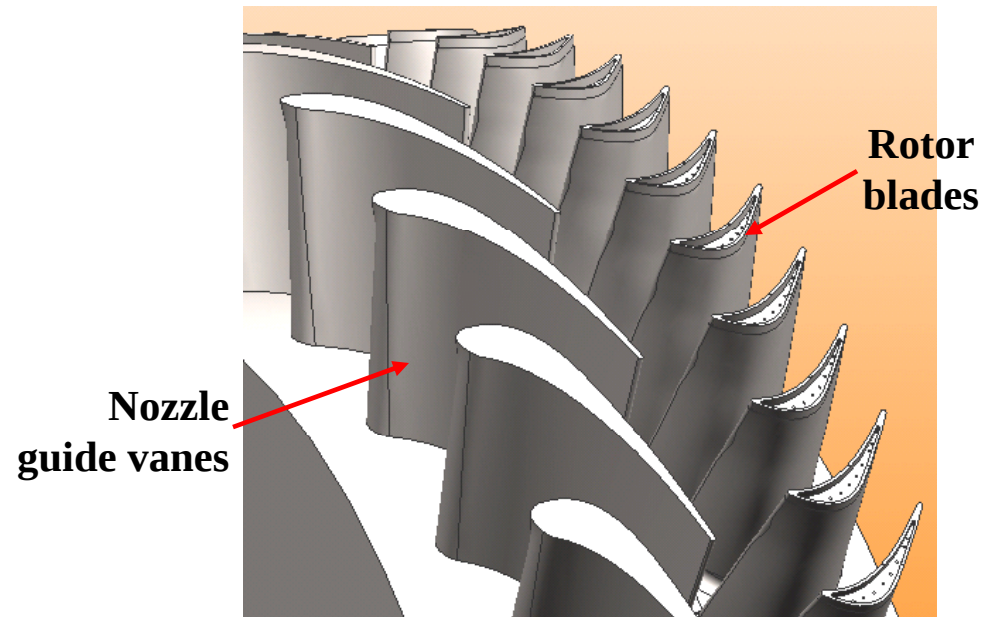
Types of Axial Turbines

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Axial turbines can be single stage or multistage



Multistage
axial turbine



Single stage
turbine

..... but the basic design principles remain the same

Types of Axial Turbines

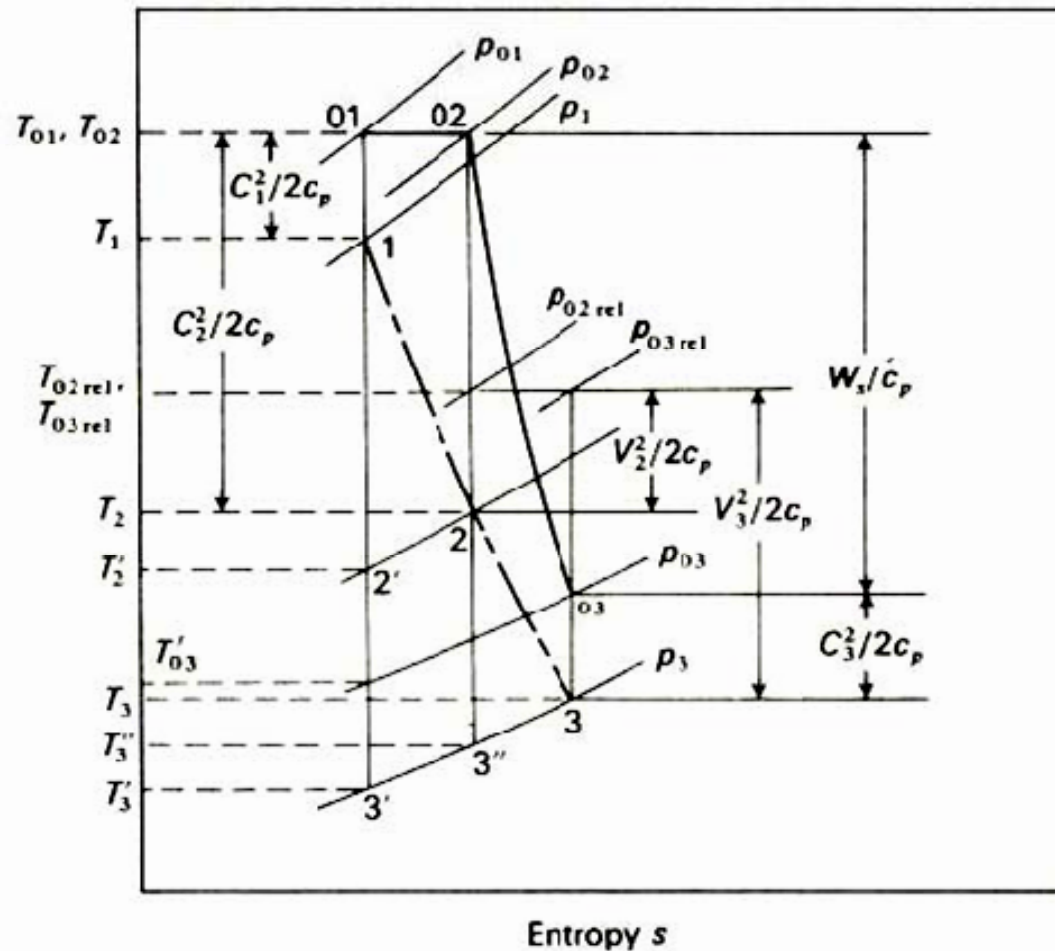
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Axial turbines can also be classified as:

- Subsonic turbines
- Transonic turbines
- Supersonic turbines

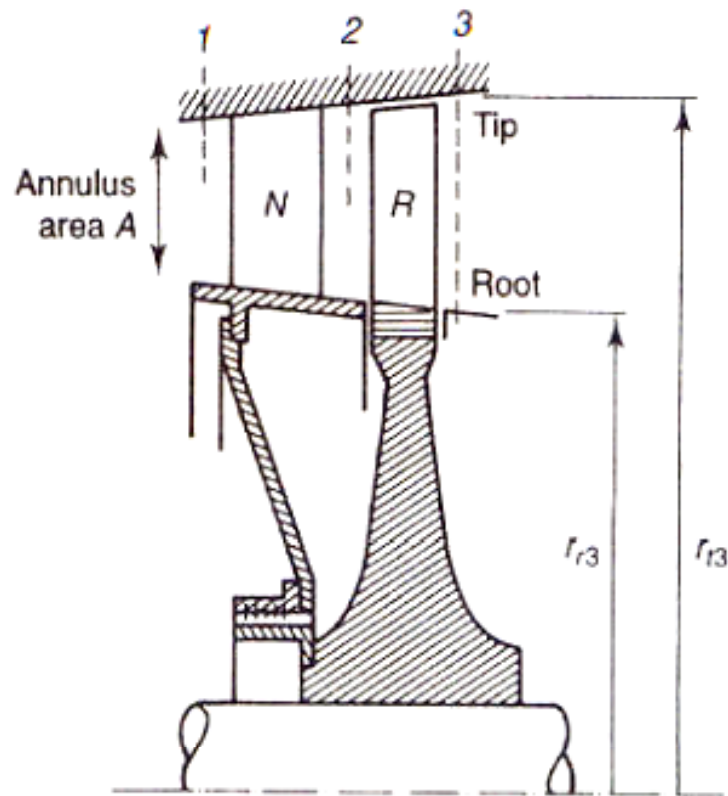
Expansion Process on T-S Diagram

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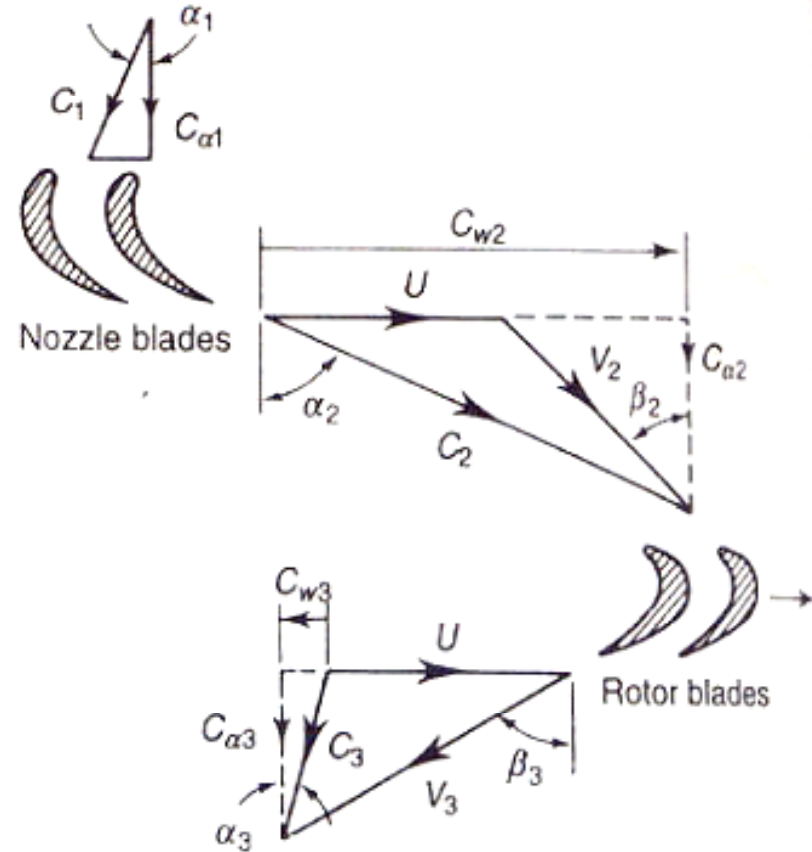


Velocity Triangles

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Axial flow turbine stage



Euler Turbine Equation

In a turbine, the fluid does work on the rotor.

Hence, the specific work is given by

$$\dot{W}_t = \frac{W_t}{\dot{m}} = (U_2 C_{w2} - U_3 C_{w3}) > 0 \quad \text{Watt . s /kg}$$

Specific work can also be related to the change in total enthalpy:

$$\dot{W}_t = (W_t / \dot{m}) = (U_2 C_{w2} - U_3 C_{w3}) = (h_{02} - h_{03}) \quad \text{Watts s /kg}$$

Turbine Efficiency

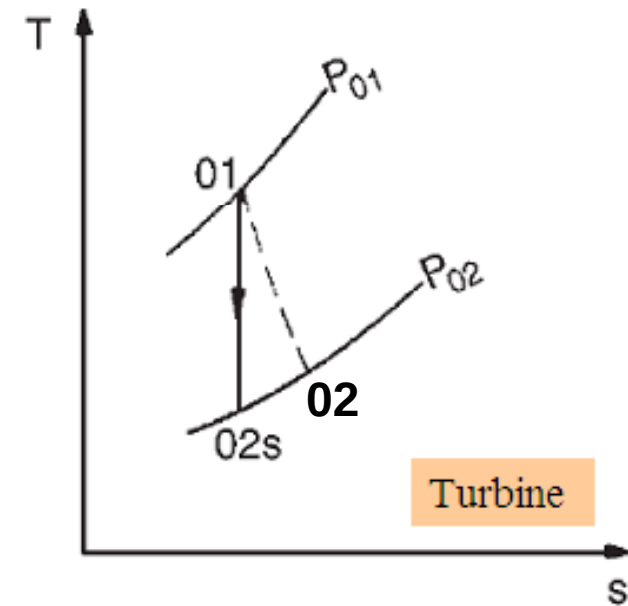
$$\eta_{isen} = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}}$$

$$= \frac{c_p (T_{01} - T_{02})}{c_p (T_{01} - T_{02s})}$$

$$(T_{01} - T_{02}) = \eta_t (T_{01} - T_{02s}) = \eta_t T_{01} \left(1 - \frac{T_{02s}}{T_{01}} \right)$$

$$\therefore \frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p_{01}} \right)^{(\gamma-1)/\gamma}$$

$$T_{01} - T_{02} = \eta_t T_{01} \left[1 - \left(\frac{1}{p_{01}/p_{02}} \right)^{(\gamma-1)/\gamma} \right]$$

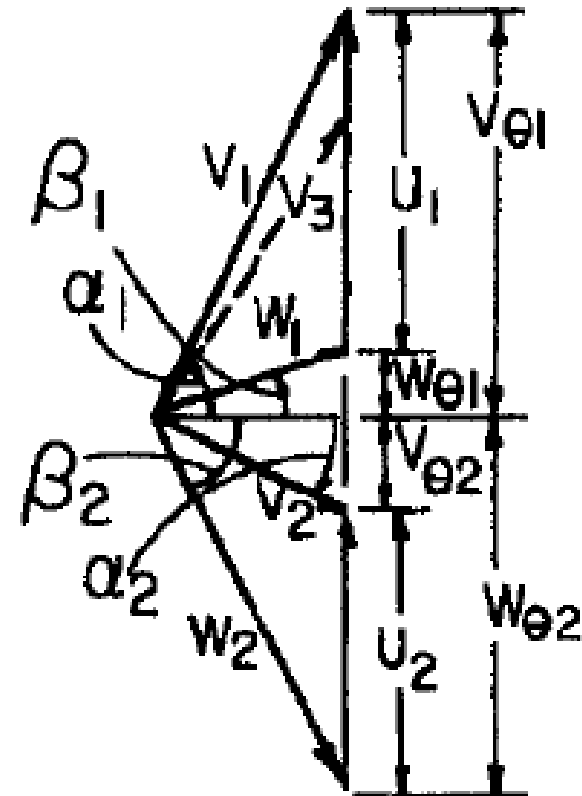
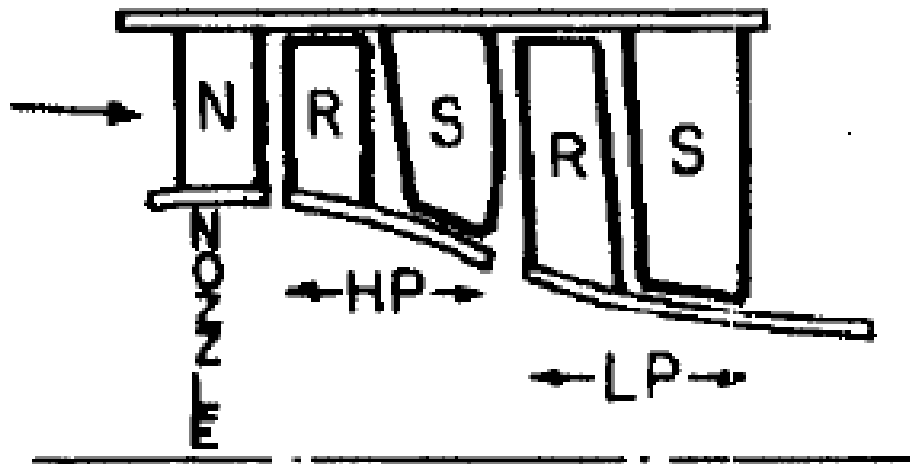


Compression process
on T-s diagram

Flow in Axial Turbine Stage

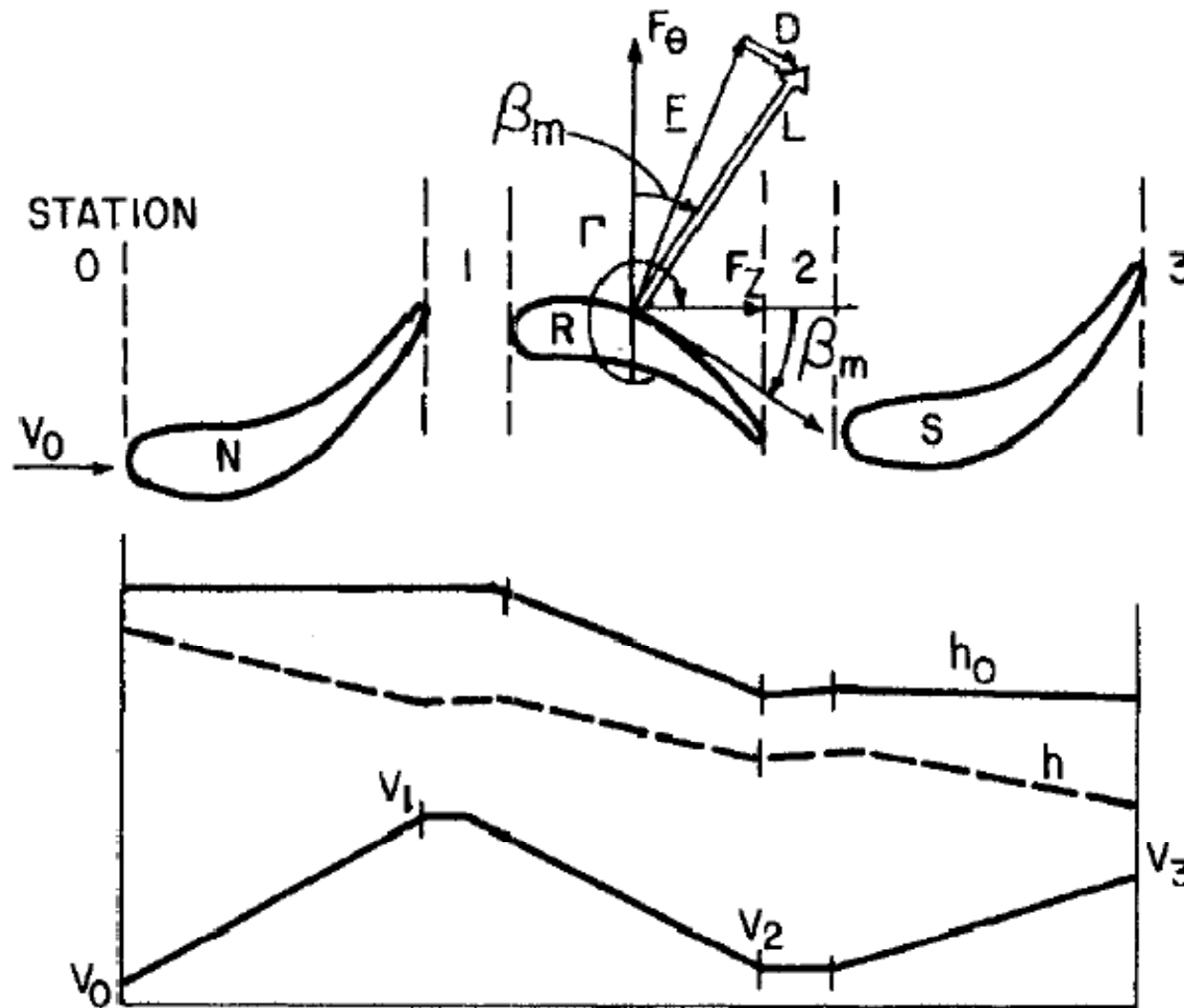
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Now consider a $1\frac{1}{2}$ stage turbine with the following nomenclature



Flow in Axial Turbine Stage

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Flow in Axial Turbine Stage

Using the above nomenclature

$$R = \frac{V_z}{2U} (\tan \beta_2 + \tan \beta_1) = (\tan \beta_m) \frac{V_z}{U}$$

Since $V_z \tan \alpha_1 = V_z \tan \beta_1 + U$

we get $\psi = 2(\phi \tan \beta_1 - R)$ for repeating stages with AVR = 1

Temperature and pressure ratios can be expressed in terms of reaction, R

$$\frac{T_{o2}}{T_{o1}} = 1 - \frac{(\gamma - 1) M_b^2}{1 + \frac{\gamma - 1}{2} M_1^2} \left[2(\phi \tan \beta_1 - R) \right]$$

$$\frac{P_{o2}}{P_{o1}} = \left[1 - \frac{(\gamma - 1) M_b^2}{\eta_t \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)} \left\{ 2(\phi \tan \beta_1 - R) \right\} \right]^{\gamma/\gamma - 1}$$

Impulse and Reaction Turbine Stages

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- Hence, the low and zero reaction turbines have a higher temperature drop (work output) but lower temperature ratio and pressure ratio than does a corresponding high reaction turbine for the same mass flow, blade speed, and relative inlet flow angle.
- For repeating stages having axial exit flow with no swirl ($\phi \tan \beta_3 = 1$), i.e. repeating stages with constant axial velocity and axial exhaust,

$$\psi = 2(1 - R)$$

- Therefore, a zero reaction turbine (~ impulse turbine) produces twice as much work as a 50% reaction turbine.**

Impulse and Reaction Turbine Stages

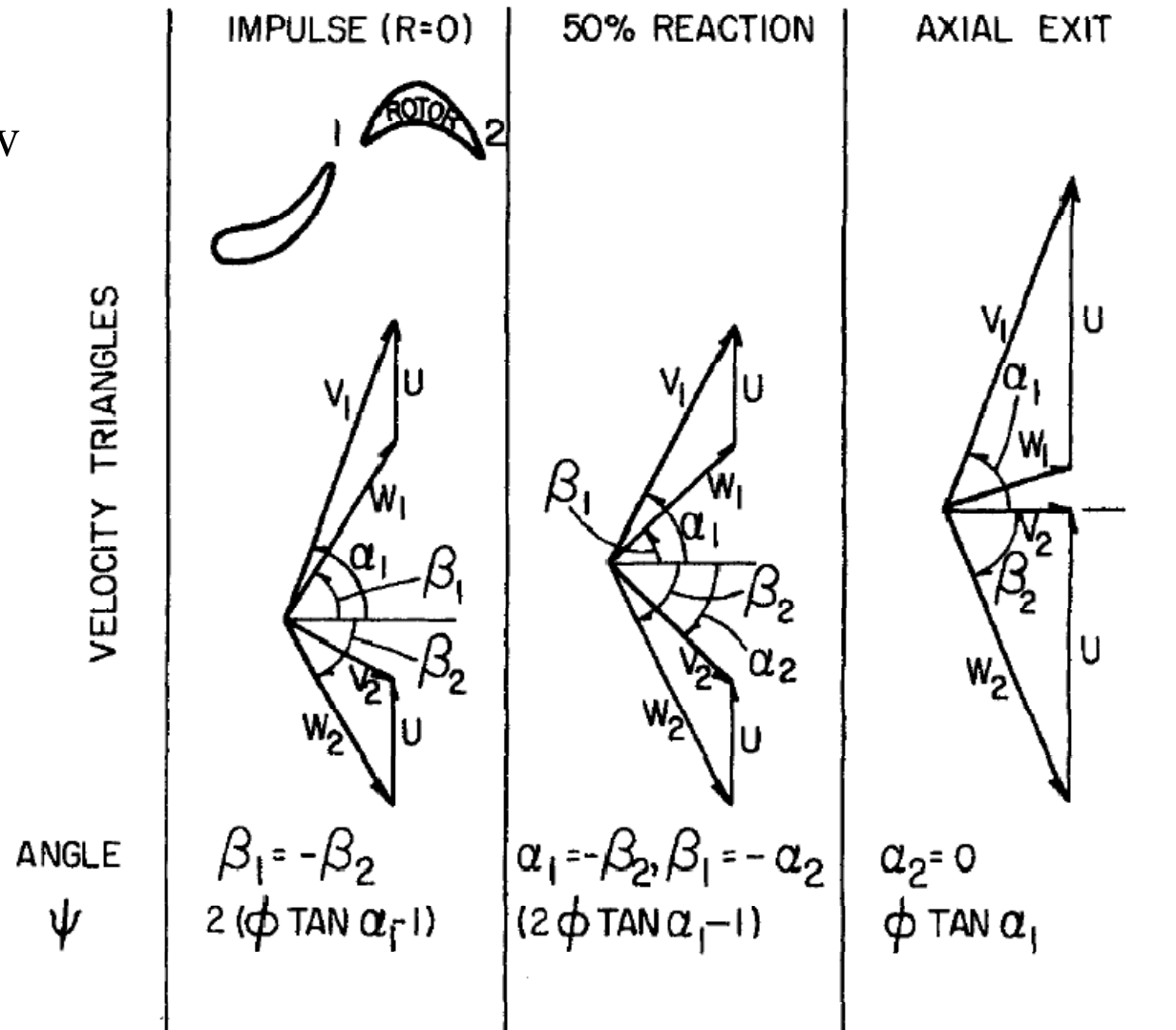
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- ❑ Temperature drop is higher for low/zero reaction turbines, hence the subsequent blade rows have lower temperatures and lower cooling requirements.
- ❑ Rotor faces *relative* stagnation temperature and stator faces *absolute* stagnation temperature. Both these quantities are **low for an impulse rotor and stator** with consequent lower cooling requirement.
- ❑ The **disadvantage** of an impulse stage is that all the acceleration occurs in the stator passage, producing increased losses.
- ❑ **Thus, an impulse stage has lower efficiency than a corresponding reaction turbine.**

Impulse and Reaction Turbine Stages

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- Degree of reaction varies from a high value at the tip to a low value at the hub.
- The degree of reaction during mean line design should be chosen such that it does not become negative at the hub causing the flow to decelerate.
- A decelerating flow will give rise to lower work output.



Impulse and Reaction Turbine Stages

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Consider an impulse stage ($R \simeq 0$).

$$W_1 = W_2, h_1 = h_2, (h_{o1})_R = (h_{o2})_R, \text{ and } \beta_1 = -\beta_2$$

and all the static pressure drop occurs in the nozzle or the stator and all the stagnation pressure drop occurs in the rotor. For AVR = 1,

$$\psi = 2(\phi \tan \alpha_1 - 1)$$

For a 50% reaction turbine, the velocity triangles are symmetrical and

$$\psi = (2\phi \tan \alpha_1 - 1)$$

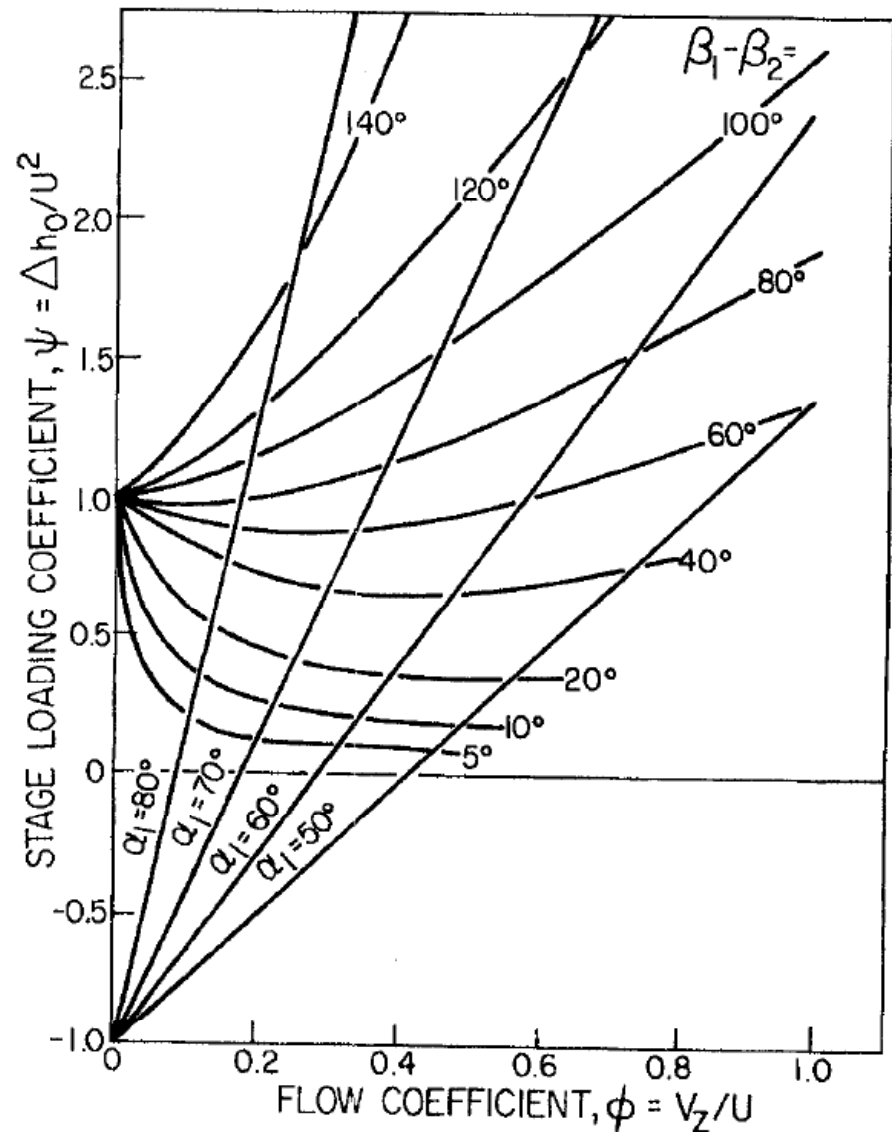
Impulse and Reaction Turbine Stages

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- At a given α_1 , ψ increases linearly with ϕ .
- At a fixed ϕ , an increase in α_1 results in increased ψ .
- For example, at $\phi=0.5$, increasing α_1 from 50 to 70 increases ψ from 0.2 to 1.75 !
- Current design trend is to use high α_1 nozzles.
- High α_1 increases W_1 for a given blade speed. Flow may become supersonic limiting the mass flow rate.

Plot of the equation

$$\psi = (2\phi \tan \alpha_1 - 1)$$
 for a 50% reaction turbine



Turbine Efficiency

□ Apart from total-to-total and total-to-static efficiencies, another useful performance measure of a rotor is the efficiency based on the kinetic energy at the inlet to the rotor.

$$\eta_{KE} = \frac{h_{o1} - h_{o2}}{V_1^2/2}$$

□ This represents how well the rotor converts potential and kinetic energy into mechanical power.

□ KE at exit can not be zero, hence $\eta_{KE}=1$ can not be achieved in practice.

□ For isentropic turbines, the optimum values of η_{KE} are:

$$\begin{aligned} \eta_{KE} &= \frac{\psi U^2}{V_1^2/2} = \frac{4(\phi \tan \alpha_1 - 1)}{\sigma^2} \quad (\text{Impulse}) \\ &= 2 \frac{(2\phi \tan \alpha_1 - 1)}{\sigma^2} \quad (50\% \text{ reaction stage}) \end{aligned}$$

where $\sigma = V_1/U$ is the velocity ratio to be optimised

Turbine Efficiency

- For a reversible **impulse** stage

$$\eta_{KE} = 4 \frac{(\sigma \sin \alpha_1 - 1)}{\sigma^2}$$

For a given α_1 , we have $\partial \eta_{KE} / \partial \sigma = 0$ when $\sigma = V_1 / U = 2 / \sin \alpha_1$.

- Similarly, for a reversible **50% reaction** stage

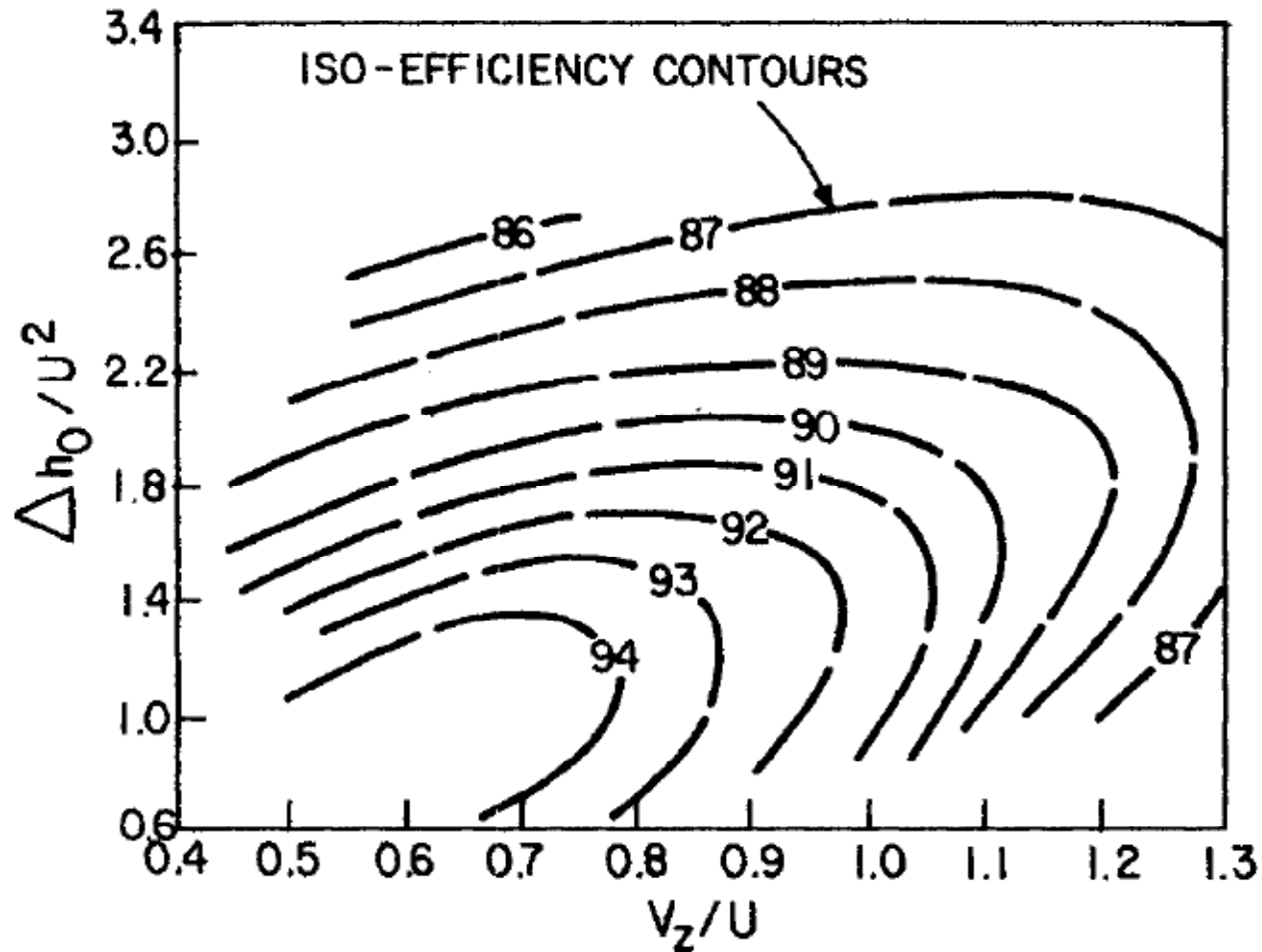
$$\eta_{KE} = \frac{2(2\sigma \sin \alpha_1 - 1)}{\sigma^2}$$

Therefore, for this stage, η_{KE} is maximum when $\sigma = 1 / \sin \alpha_1$

- Because α_1 is usually large, the best η_{KE} is achieved when V_1 / U is close to unity for a 50% reaction turbine and close to 2 for an impulse turbine.

Choice of Blade Loading

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Data compiled and correlated by Kacker and Okapuu (1982) from 100 sets of data from 33 turbines

Choice of Blade Loading

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- ❑ Efficiency of a turbine depends on the loading coefficient and the flow coefficient.
- ❑ Loading coefficient influences pressure gradient in the passage, and this increases losses.
- ❑ Higher flow coefficient (higher mass flow rate) results in higher pressure drop and corresponding losses also increase.
- ❑ Hence, highest efficiencies are achieved at low loading and low flow coefficient.

Degree of Reaction

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Degree of Reaction:

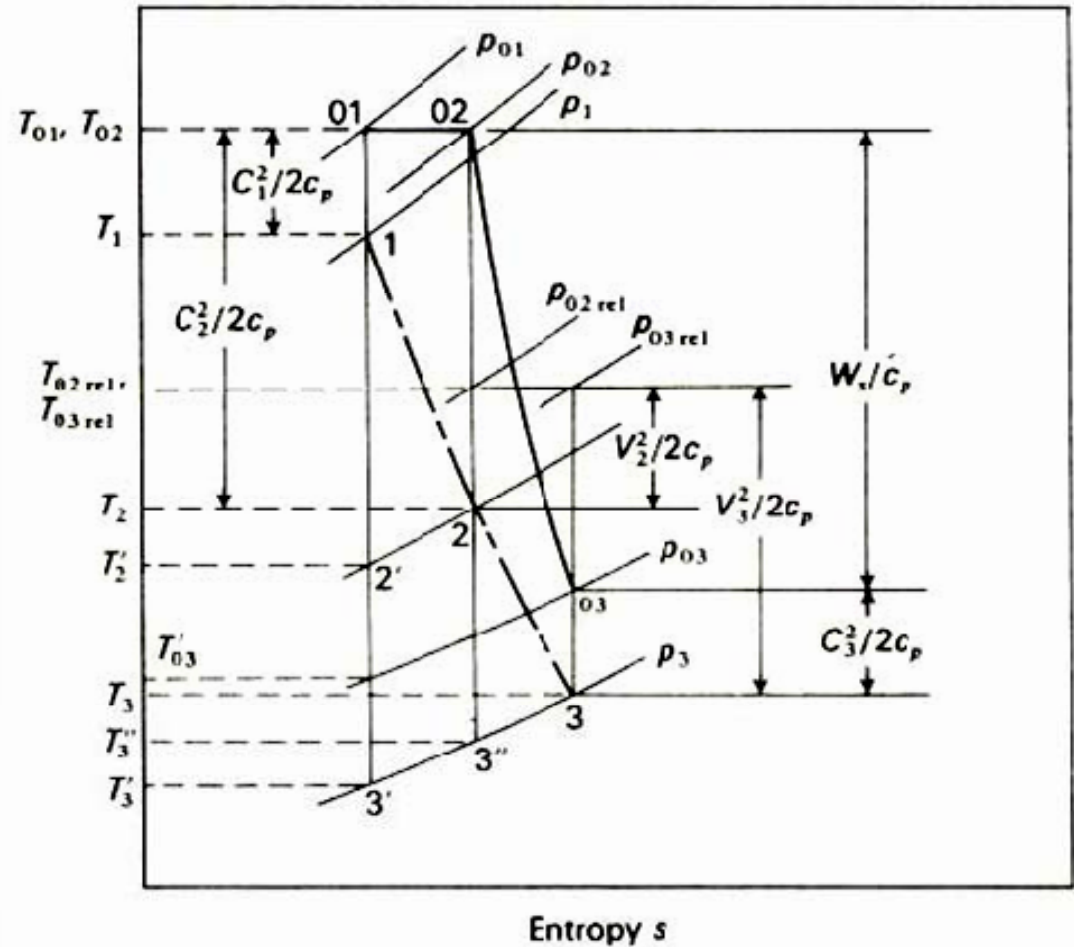
$$\begin{aligned}
 R &= \frac{T_2 - T_3}{T_1 - T_3} \\
 &= \frac{C_a}{2U} (\tan \beta_3 - \tan \beta_2) \\
 &= \frac{1}{2} + 2 \frac{C_a}{U} (\tan \beta_3 - \tan \alpha_2)
 \end{aligned}$$

Also,

$$\psi = 2 \Phi (\tan \beta_2 + \tan \beta_3)$$

$$R = \frac{1}{2} \Phi (\tan \beta_3 - \tan \beta_2)$$

$$\text{where } \Phi = C_a / U$$



Degree of Reaction

Degree of Reaction, R , is defined as the ratio of the static enthalpy drop in the rotor to the static enthalpy drop in the whole stage.

$$R = \frac{\Delta T_A}{\Delta T_A + \Delta T_B} \quad \left| \quad \begin{array}{l} \Delta T_A : \text{Static temperature drop in the rotor} \\ \Delta T_B : \text{Static temperature drop in the stator} \end{array} \right.$$

Since $c_p \Delta T_{0s} \approx c_p \Delta T_s$

$$W = c_p (\Delta T_A + \Delta T_B) = c_p \Delta T_s = c_p (T_1 - T_3)$$

$$= c_p (T_{01} - T_{03}) = UC_a (\tan \beta_2 + \tan \beta_3)$$

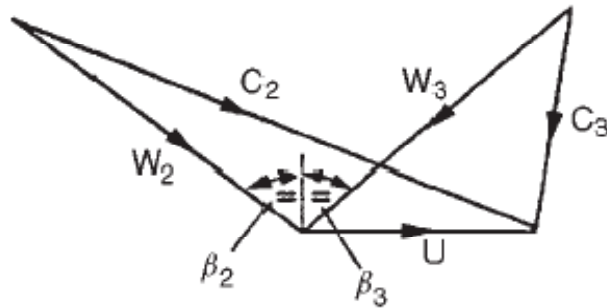
Also, $c_p \Delta T_A = c_p (T_2 - T_3) = \frac{1}{2} C_a^2 (\tan^2 \beta_3 - \tan^2 \beta_2)$

It can be shown that

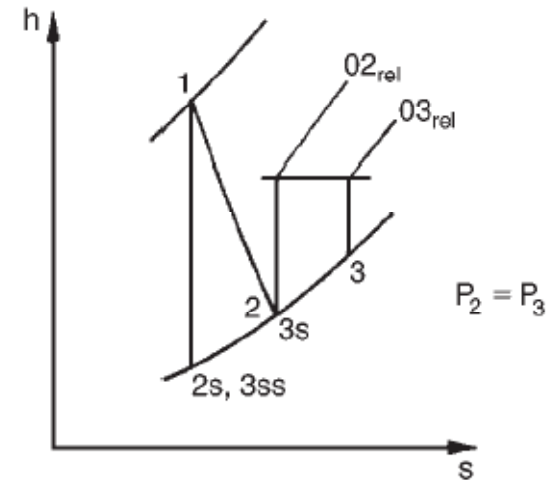
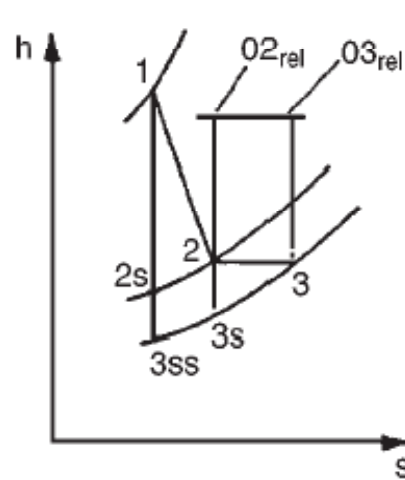
$$R = \frac{C_a}{2U} (\tan \beta_3 - \tan \beta_2) = \frac{1}{2} + 2 \frac{C_a}{U} (\tan \beta_3 - \tan \alpha_2)$$

Degree of Reaction

Zero Reaction Stage



Velocity diagram and Mollier diagram for a zero reaction turbine stage

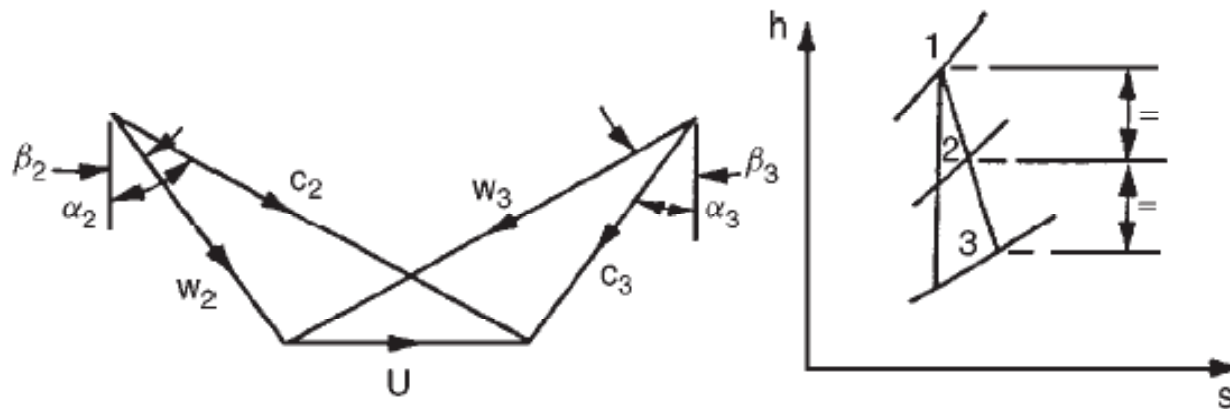


Mollier diagram for an impulse turbine stage

From the definition of reaction, when $R = 0$, we have $h_2 = h_3$ and $\beta_3 = \beta_2$. As $h_{02\text{rel}} = h_{03\text{rel}}$ and $h_2 = h_3$ for $R = 0$ it must follow, therefore, that $W_2 = W_3$. Because of irreversibility, there is a *pressure drop* through the rotor row. The *zero reaction* stage is *not* the same as an *impulse* stage; in the latter case, by definition, there is no pressure drop through the rotor. The Mollier diagram for an impulse stage is also shown, where it is seen that the enthalpy *increases*.

Degree of Reaction

50% Reaction Stage



Velocity diagram and Mollier diagram for a 50% reaction turbine stage

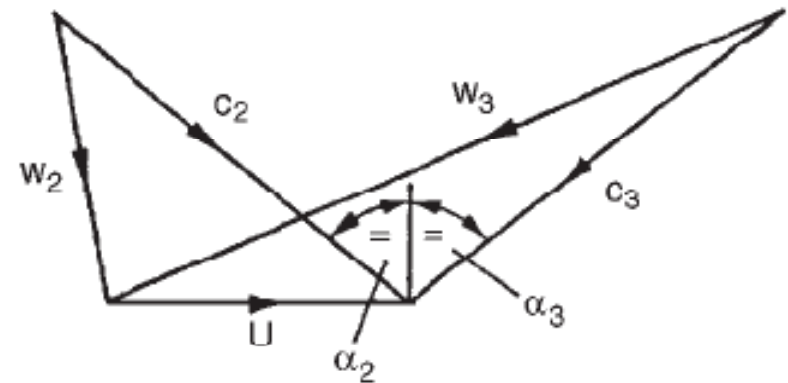
The combined velocity diagram for this case is symmetrical since $\beta_3 = \alpha_2$. Because of the symmetry it is obvious that $\beta_2 = \alpha_3$, also. With $R = 1/2$, the enthalpy drop in the nozzle row equals the enthalpy drop in the rotor, or $h_1 - h_2 = h_2 - h_3$.

Degree of Reaction

100% Reaction Stage – Diffusion within Blade Rows

Any diffusion of the flow through turbine blade rows is particularly undesirable and must be avoided. This is because the adverse pressure gradient (arising from the flow diffusion) coupled with large fluid deflection (usual in turbine blade rows), increases the chances of boundary-layer separation causing large scale losses. A compressor blade row, on the other hand, is designed to cause the fluid pressure to rise in the direction of flow, i.e. an *adverse* pressure gradient. The magnitude of this gradient is strictly controlled in a compressor, mainly by having a fairly limited amount of fluid deflection in each blade row. Substituting $\tan \beta_3 = \tan \alpha_3 + U/C_x$ in the earlier equation, we get $R = 1 + \frac{c_x}{2U}(\tan \alpha_3 - \tan \alpha_2)$

Thus, when $\alpha_3 = \alpha_2$, the reaction is unity. It will be apparent that if R exceeds unity, then $C_2 < C_1$ (i.e. nozzle flow *diffusion*).



Velocity diagram for a 100% reaction turbine stage

Choice of Degree of Reaction

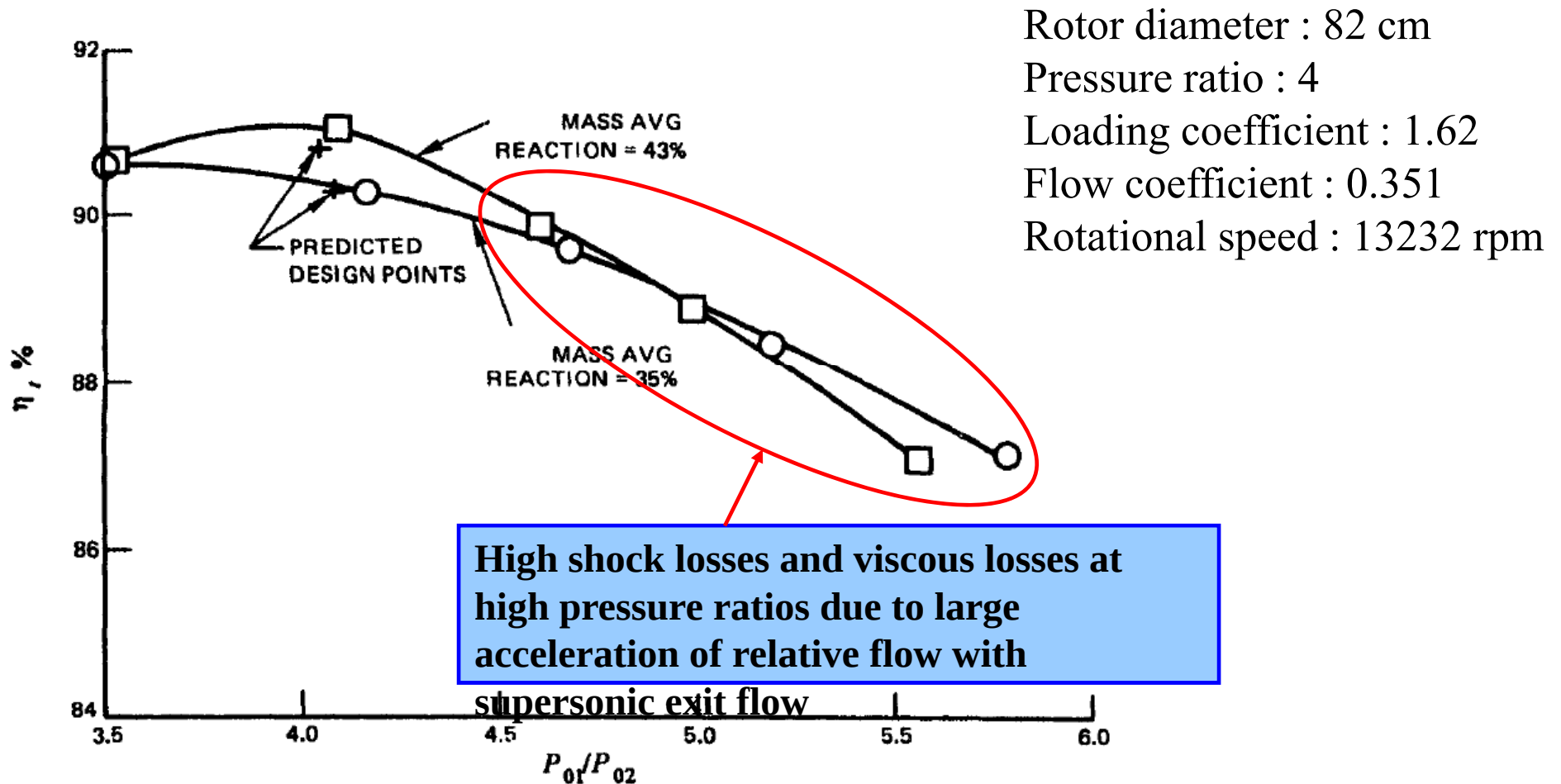
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- ❑ Consider $C_{a3}=C_{a2}$ and $C_3=C_1$  repeating stages.
- ❑ The blade speed U is limited by stressing considerations.
- ❑ Overall temperature drop is fixed from cycle calculations, but the choice of R is infinite
- ❑ The designer can choose one or two stages of large ψ or a larger number of stages of smaller ψ .
- ❑ Any turbine for a gas turbine plant is a relatively low pressure ratio (20-40:1) machine compared to steam turbine plants (1000:1).
- ❑ Impulse stages are desirable owing to low rotor tip leakage losses and hence these are employed in the high pressure stages of steam turbines. Reaction stages result in excessive tip leakage.
- ❑ Impulse stages are not employed in gas turbines because the pressure levels are relatively too low.

Choice of Reaction

- ❑ For the same total enthalpy drop ($h_{01} - h_{02}$) in a stage, an optimum reaction turbine requires a higher blade speed than an optimum impulse turbine.
- ❑ If the blade speed is fixed, it can be shown that a reaction turbine requires more stages (for the same total output power) than an impulse turbine.

Effect of Reaction on Efficiency



Data from Turbine for Energy Efficient Engine
[Thulin et al (1982) and Leach (1983)]

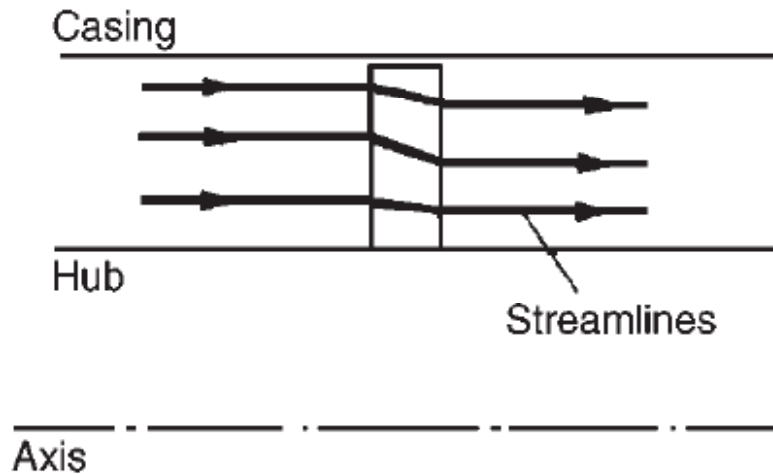
3-D Flows in Turbine Blade Rows

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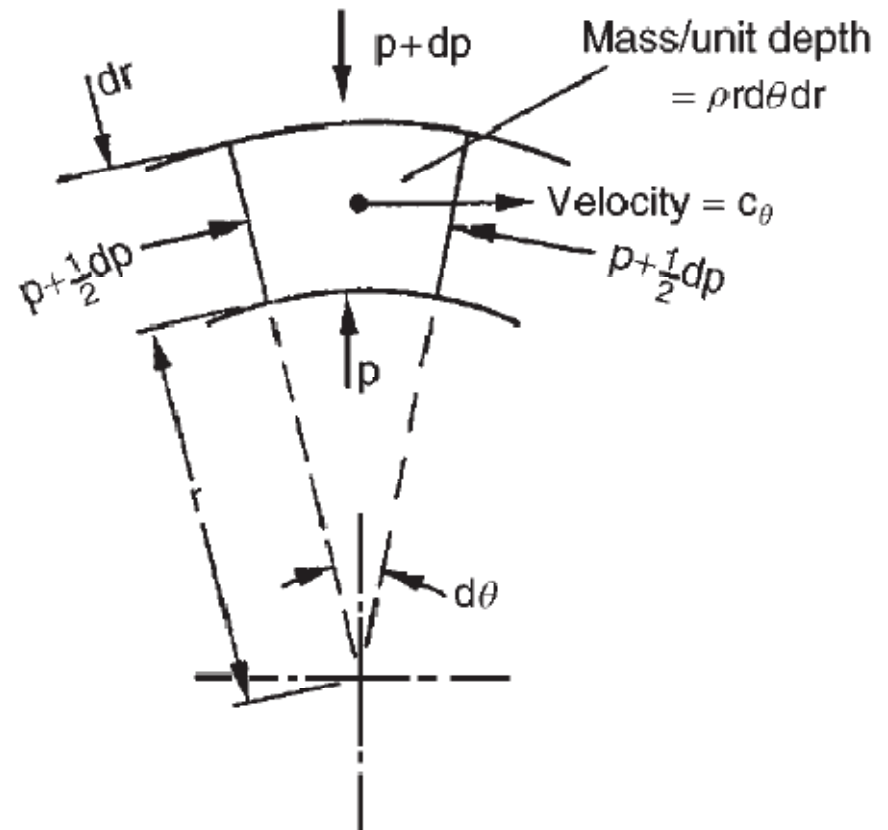
Some of the three dimensional inviscid effects are due to:

1. Compressibility, and radial density and pressure gradients
2. Radial variation in blade thickness and geometry
3. Presence of finite hub and annulus walls, annulus area changes, flaring, curvature and rotation
4. Radially varying work input or output
5. Presence of two phase flow, and coolant injection
6. Radial component of blade force and the effects of blade skew, sweep, lean and twist
7. Leakage flow due to tip clearance and axial gaps
8. Nonuniform inlet flow and presence of upstream and downstream blade rows
9. Mixed-flow (subsonic, supersonic, and transonic flow) regions along the blade height with shock–boundary-layer interaction
10. Secondary flow caused by inlet velocity/stagnation pressure gradient and flow turning

Radial Equilibrium of Fluid Element



**Radial equilibrium flow through
a rotor blade row**



**A fluid element in radial
equilibrium ($C_r = 0$)**

Radial Equilibrium Equation

The basic assumption of the radial equilibrium is that the radial velocity C_r is zero at entry and exit from a blade row.

Starting from the equation of motion in cylindrical coordinates, the variation in C_r is written as

$$C_r \frac{\partial C_r}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_r}{\partial \theta} + C_x \frac{\partial C_r}{\partial x} - \frac{C_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}$$

If there are large number of blades, then variations in θ direction may be neglected.

$$C_r \frac{\partial C_r}{\partial r} + C_x \frac{\partial C_r}{\partial x} - \frac{C_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}$$

Further, if there is no component of velocity in the radial direction, i.e. if there is radial equilibrium, then $C_r = 0$, and the above equation reduces to

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{C_\theta^2}{r}$$



Radial equilibrium equation indicating that the pressure forces on the fluid particles are balanced by the centrifugal forces

Axial Velocity Distribution

For incompressible flow: $p_0 = p + \frac{1}{2} \rho (C_x^2 + C_\theta^2)$

$$\begin{aligned} \text{and } \frac{1}{\rho} \frac{dp_0}{dr} &= \frac{1}{\rho} \frac{dp}{dr} + C_x \frac{dC_x}{dr} + C_\theta \frac{dC_\theta}{dr} \\ &= \frac{C_\theta^2}{r} + C_x \frac{dC_x}{dr} + C_\theta \frac{dC_\theta}{dr} \\ &= C_x \frac{dC_x}{dr} + \frac{C_\theta}{r} \frac{d}{dr} (r \cdot C_\theta) \end{aligned}$$

If the total pressure is assumed constant along the radius, then

$$C_x \frac{dC_x}{dr} + \frac{C_\theta}{r} \frac{d}{dr} (r \cdot C_\theta) = 0$$

or $\frac{d}{dr} (C_x^2) + \frac{1}{r^2} \frac{d}{dr} (r \cdot C_\theta)^2 = 0 \Rightarrow$ Gives variation of axial velocity with radius

Axial Velocity Distribution

Similarly, for compressible flow: $h_0 = h + \frac{1}{2}(C_x^2 + C_\theta^2)$

$$\frac{dh_0}{dr} = \frac{dh}{\partial r} + C_x \frac{dC_x}{dr} + C_\theta \frac{dC_\theta}{dr}$$

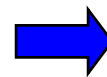
But $T \frac{ds}{dr} = \frac{dh}{\partial r} + \frac{1}{\rho} \frac{dp}{dr}$

$$\begin{aligned} \frac{dh_0}{dr} - T \frac{ds}{dr} &= \frac{1}{\rho} \frac{dp}{dr} + C_x \frac{dC_x}{dr} + C_\theta \frac{dC_\theta}{dr} \\ &= C_x \frac{dC_x}{dr} + \frac{C_\theta}{r} \frac{d}{dr}(r \cdot C_\theta) \end{aligned}$$

If $\frac{dh_0}{dr} = 0$ and $T \frac{ds}{\partial r} = 0$

Then $C_x \frac{dC_x}{dr} + \frac{C_\theta}{r} \frac{d}{dr}(r \cdot C_\theta) = 0$

or $\boxed{\frac{d}{dr}(C_x^2) + \frac{1}{r^2} \frac{d}{dr}(r \cdot C_\theta)^2 = 0}$



Gives variation of axial velocity with radius

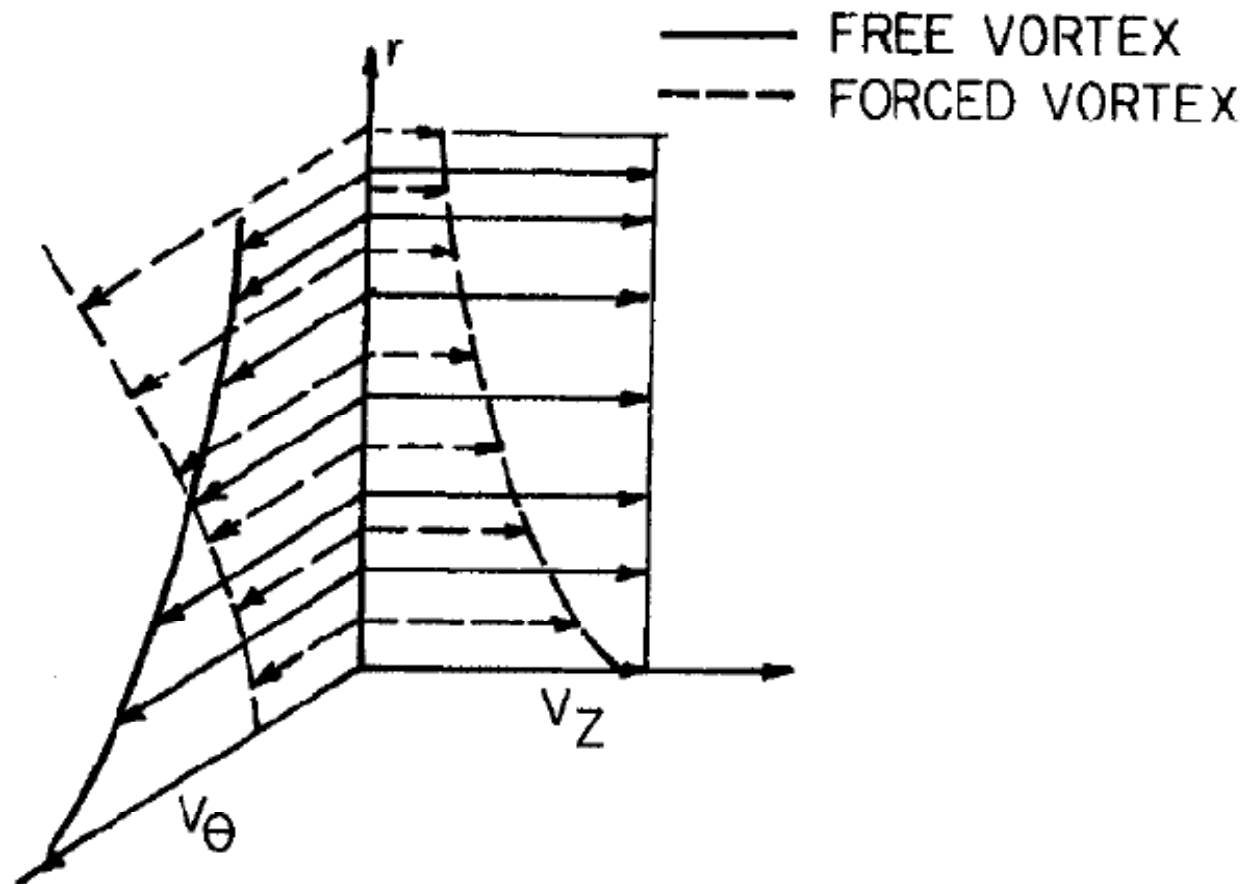
Types of Whirl Distribution

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The whirl (vortex) distributions normally used in compressor design practice are:

- Free vortex $r C_{\theta} = \text{constant}$
 - Forced vortex $C_{\theta} / r = \text{constant}$
 - Constant reaction $R = \text{constant}$
 - Exponential $C_{\theta 1} = a - b/r$ (after stator)
 $C_{\theta 2} = a + b/r$ (after rotor)
-
- Free vortex whirl distribution results in highly twisted blades and is not advisable for blades of small height.
 - The current design practice for transonic compressors is to use constant pressure ratio across the span.

Free and Forced Vortex Velocity Distribution



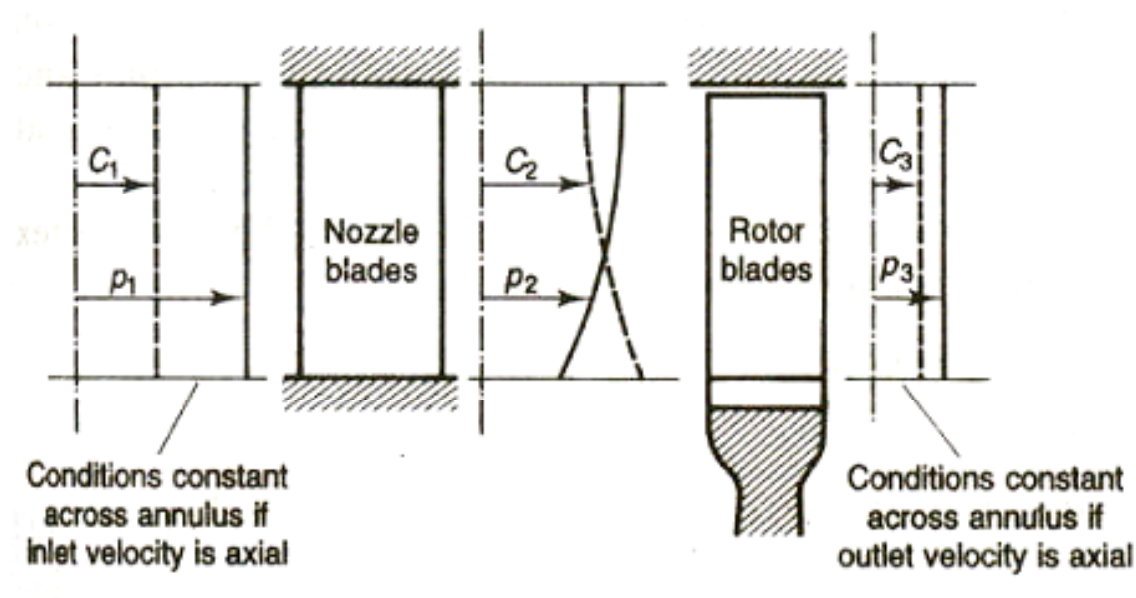
Free Vortex Design

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- For free vortex design
 - a) The stagnation enthalpy h_o is constant over the annulus (i.e. $dh_o/dr = 0$),
 - b) The axial velocity is constant over the annulus,
 - c) The whirl velocity is inversely proportional to the radius,
- Then the condition for radial equilibrium of the fluid elements, is satisfied. A stage designed in accordance with (a), (b) and (c) is called a *free vortex stage*.

Free Vortex Design (... contd.)

- Applying this to the stage in the figure, we can see that with uniform inlet conditions to the nozzles then, since no work is done by the gas in the nozzles, h_0 must also be constant over the annulus at outlet. Thus condition (a) is fulfilled in the space between the nozzles and rotor blades.



Free Vortex Design (... contd.)

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- Furthermore, if the nozzles are designed to give $C_{a2} = \text{constant}$ and $C_{w2}r = \text{constant}$, all three conditions are fulfilled and the condition for radial equilibrium is satisfied in plane-2.
- Similarly, if the rotor blades are designed so that $C_{a3} = \text{constant}$ and $C_{w3}r = \text{constant}$, it is easy to show as follows that condition (a) will be fulfilled, and thus radial equilibrium will be achieved in plane-3 also. Writing ω for the angular velocity we have

$$W_s = U(C_{w2} + C_{w3}) = \omega(C_{w2}r + C_{w3}r) = \text{constant}$$

- But when the work done per unit mass of gas is constant over the annulus, and h_o is constant at inlet, h_o must be constant at outlet also; thus condition (a) is met.

Constant Nozzle Angle Design

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- Nozzles are designed using the relation

$$C_{a2} r^{\sin^2 \alpha_2} = \text{constant}$$

- Normally nozzle angles are greater than 60° , and quite a good approximation to the flow satisfying the equilibrium condition is obtained by designing with a constant nozzle angle and constant angular momentum, i.e. $\alpha_2 = \text{constant}$ and $C_{w2} r = \text{constant}$.
- If this approximation is made and the rotor blades are twisted to give constant angular momentum at outlet also, then, as for free vortex flow, the work output per unit mass flow is the same at all radii.

Constant Nozzle Angle Design

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- Normally nozzle angles are greater than 60° , and quite a good approximation to the flow satisfying the equilibrium condition is obtained by designing with a constant nozzle angle and constant angular momentum, i.e. $\alpha_2 = \text{constant}$ and $C_{w2}r = \text{constant}$. If this approximation is made and the rotor blades are twisted to give constant angular momentum at outlet also, then, as for free vortex flow, the work output per unit mass flow is the same at all radii.

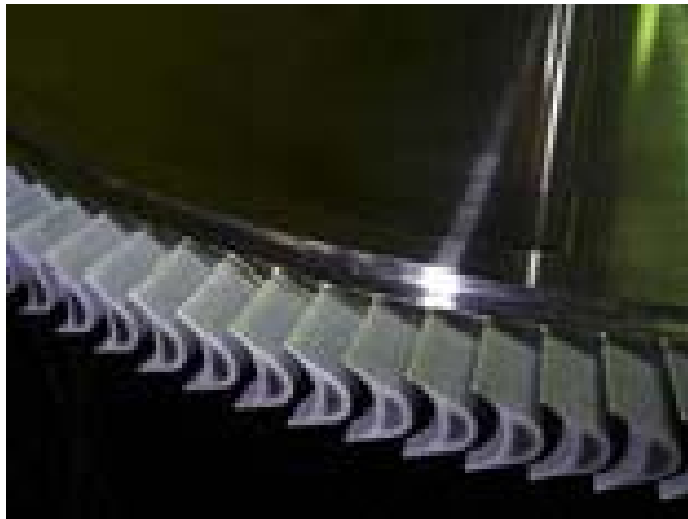
Types of Turbine Blades

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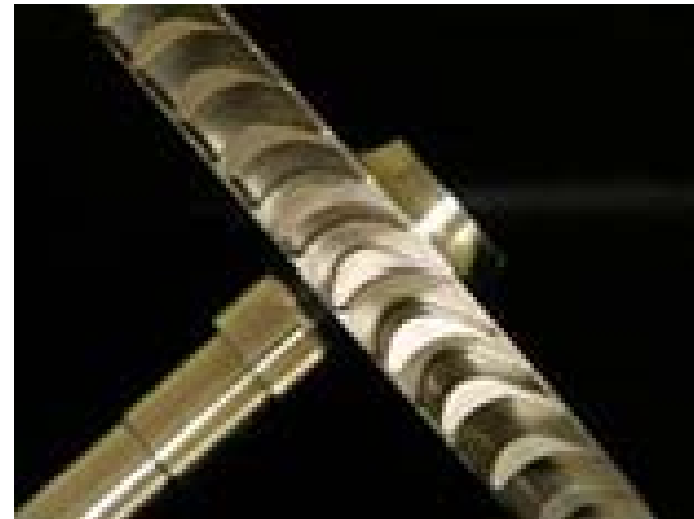


Part span snubber

Nozzle guide vanes and
rotor blades



Impulse type blades



Reaction type blades

Axial Turbine Parameters

Blade Loading Coefficient or temperature drop coefficient Ψ , which expresses the work capacity of a stage, is defined as

$$\Psi = \frac{c_p \Delta T_{0s}}{U^2/2} = \frac{U C_a (\tan \beta_2 + \tan \beta_3)}{U^2/2}$$

or
$$\Psi = \frac{2C_a}{U} (\tan \beta_2 + \tan \beta_3)$$

Degree of Reaction, R is defined as $\Rightarrow R = \frac{T_2 - T_3}{T_1 - T_3}$

if $C_{a2} = C_{a3} = C_a$ and $C_3 = C_1$

then
$$R = \frac{C_a}{2U} (\tan \beta_3 - \tan \beta_2)$$

Axial Turbine Parameters

The expressions for ψ , ϕ and R can be derived in terms of the flow angles

$$\Phi = \text{Flow coefficient} = Ca / U$$

$$\psi = 2 \Phi (\tan \beta_2 + \tan \beta_3)$$

$$R = \frac{1}{2} \Phi (\tan \beta_3 - \tan \beta_2)$$

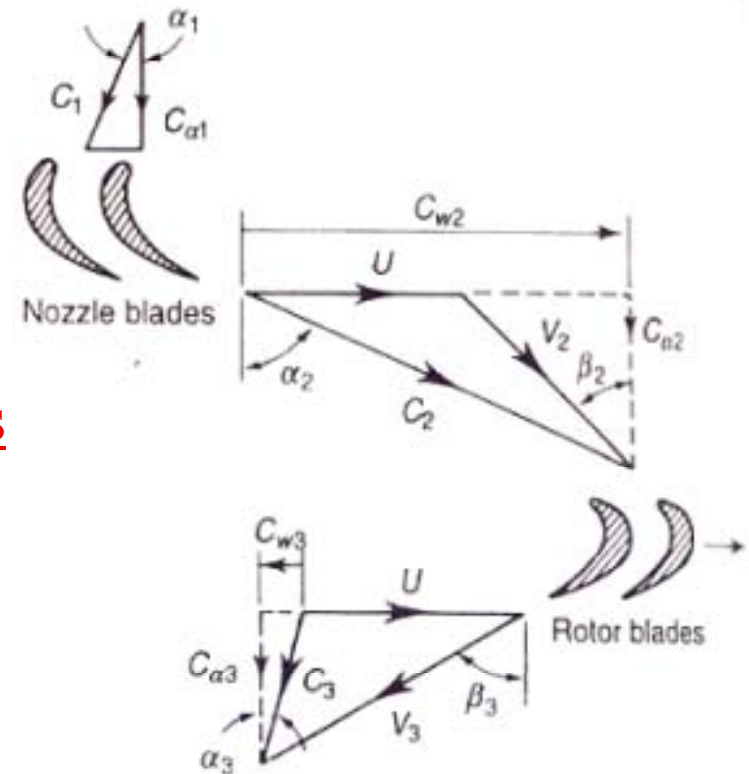
Similarly, the expressions for flow angles can be derived in terms of ψ , ϕ and R

$$\tan \beta_3 = \frac{1}{2} \Phi (\psi / 2 + 2 R)$$

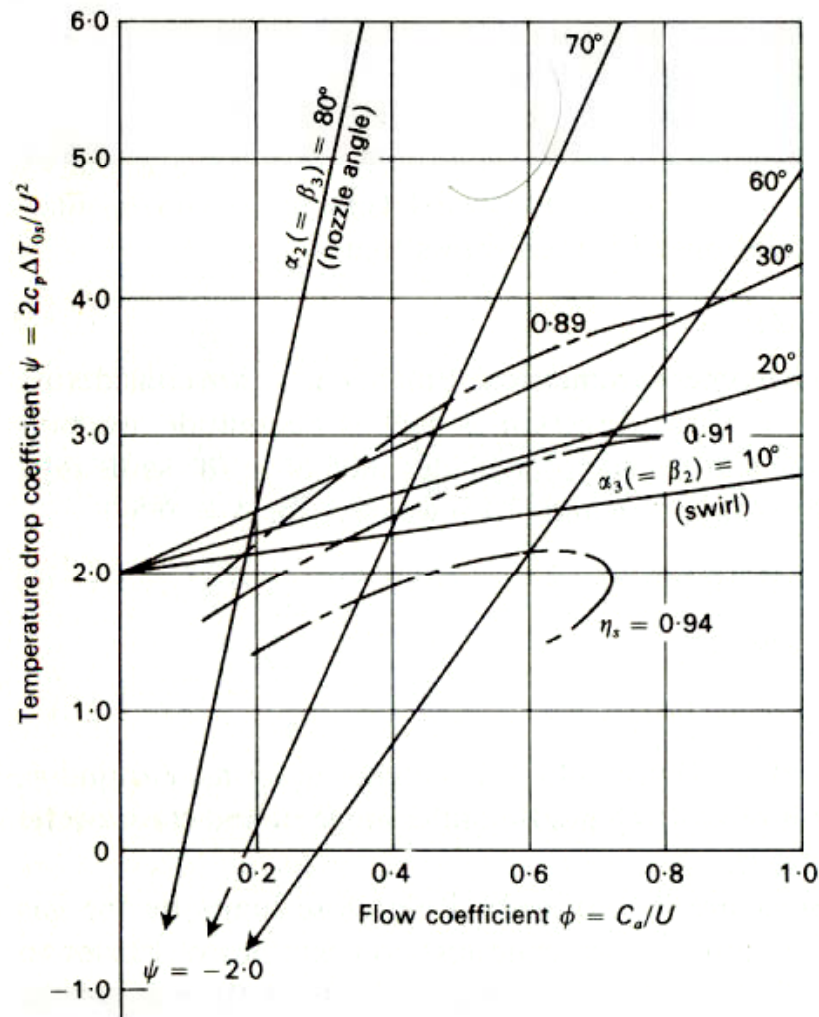
$$\tan \beta_2 = \frac{1}{2} \Phi (\psi / 2 - 2 R)$$

$$\tan \alpha_3 = \tan \beta_3 - 1/\Phi$$

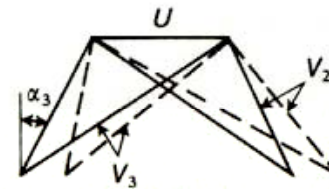
$$\tan \alpha_2 = \tan \beta_2 + 1/\Phi$$



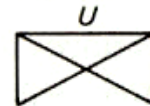
Effect of ψ and ϕ



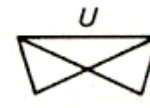
50 per cent reaction designs



$$\psi = 4.0, \phi = 1.0$$



$$\psi = 2.0, \phi = 0.6$$



$$\psi = 1.5, \phi = 0.4$$

Low Ψ means more stages for a given over all turbine output.. Low Φ means a larger turbine annulus area for a given mass flow

In industrial gas turbines, when size and weight are of little consequence and a low sfc is vital, it is sensible to design with a low Ψ and a low Φ .

For aeroengines, it is desired to keep the weight and frontal area minimum and this requires using higher values of Ψ and Φ .

For example, $\Psi = 3$ to 5 and $\Phi = 0.8$ to 1.0

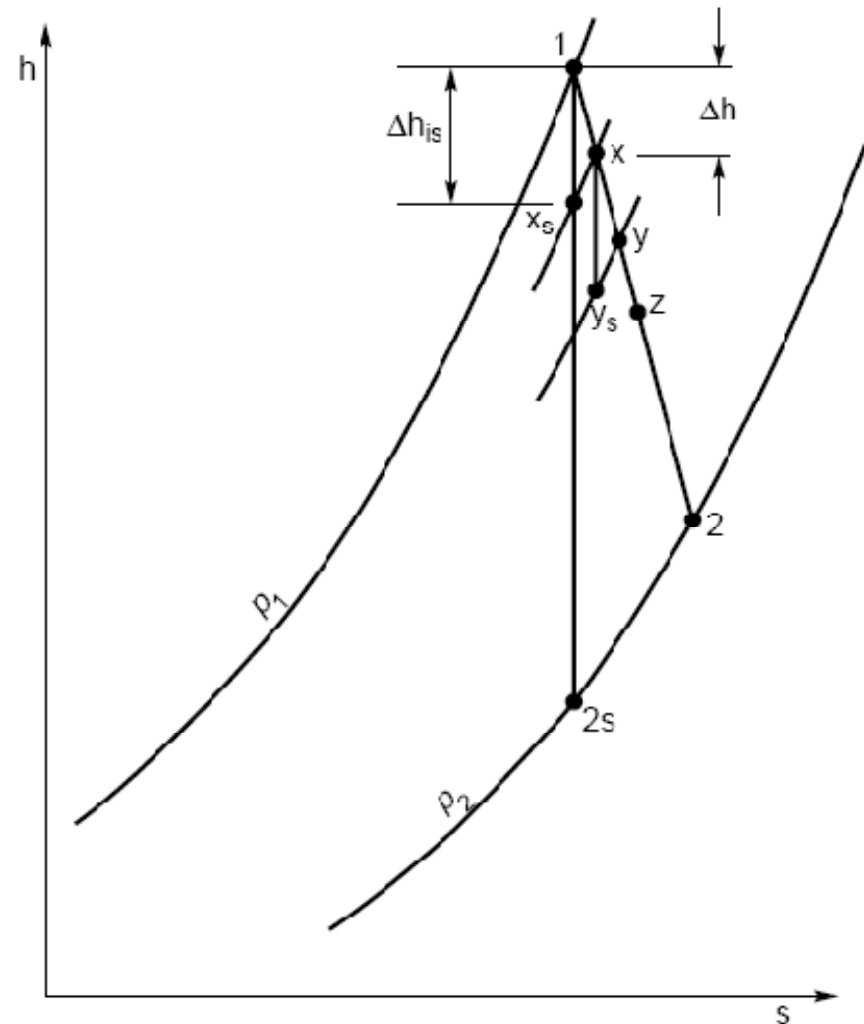
Comments on Variation of ψ and ϕ

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- ❖ Efficiency contours superimposed on Ψ - Φ plot
- ❖ Designs having a low Ψ and low Φ yield the best stage efficiencies.
- ❖ Low values of Ψ and Φ imply low gas velocities and hence reduced friction losses.
- ❖ Low Ψ means more stages for a given overall turbine output. Low Φ means a larger turbine annulus area for a given mass flow.
- ❖ In industrial gas turbine when size and weight are of little consequence and a low sfc is vital, it would be sensible to design with a low Ψ and a low Φ . Certainly in the last stage a low axial velocity and a small swirl angle α_3 are desirable to keep down the losses in the exhaust diffuser.
- ❖ For an aircraft propulsion unit, it is important to keep the weight and frontal area to a minimum and this means using higher values of Ψ and Φ . For example, $\Psi = 3 - 5$ and $\Phi = 0.8 - 1.0$

Polytropic Efficiency

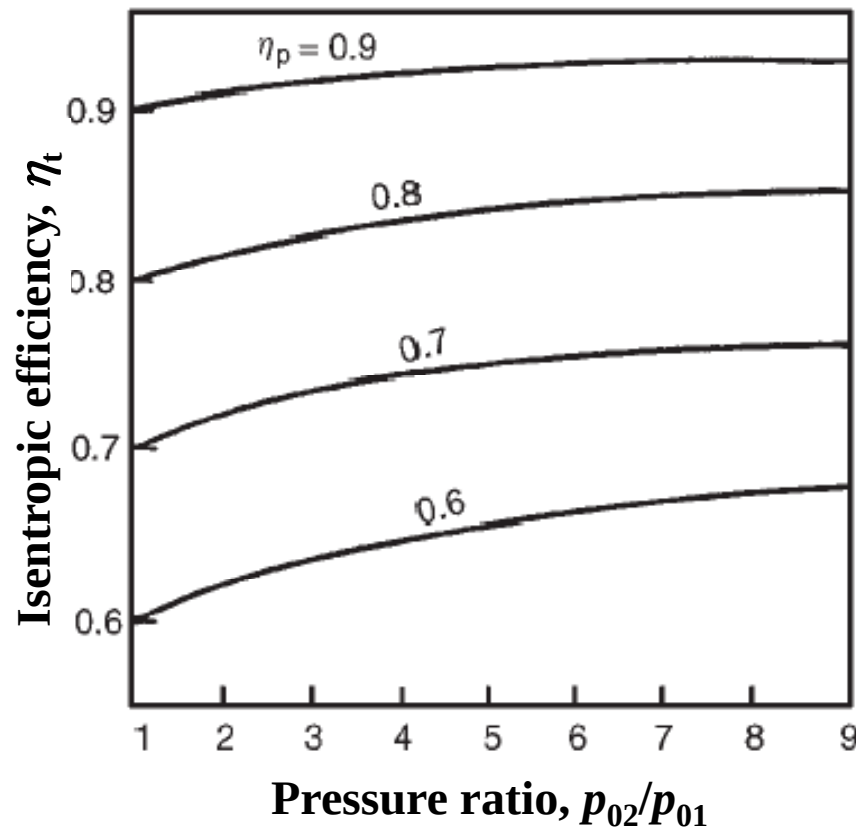
The isentropic *efficiency* described earlier, although fundamentally valid, can be misleading if used for comparing the efficiencies of turbomachines of differing pressure ratios. Now any turbomachine may be regarded as being composed of a large number of very small stages irrespective of the actual number of stages in the machine. If each small stage has the same efficiency, then the isentropic efficiency of the whole machine will be different from the small stage efficiency, the difference depending upon the pressure ratio of the machine.



Mollier diagram showing expansion process through a turbine split up into a number of small stages.

Polytropic Efficiency

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Polytropic efficiency is the efficiency of a turbine stage operating between infinitesimal pressure differential Δp . It is used in comparing the performance of two turbines having the same pressure ratio but operating at different temperature levels.

In multistage turbines, the polytropic efficiency can be used in defining the isentropic efficiency of individual stages.

$$\eta_t = \frac{1 - \left(\frac{p_{02}}{p_{01}} \right)^{\frac{\eta_p(\gamma-1)}{\gamma}}}{1 - \left(\frac{p_{02}}{p_{01}} \right)^{\frac{\gamma-1}{\gamma}}}$$

Blade Loading

❑ Blade loading primarily depends upon two parameters:

❑ Blade spacing

❑ Flow turning

Blade Spacing

Close spacing → larger number of blades → smaller force per blade
→ larger weight
→ increased frictional losses due to increased blade surfaces

Large spacing → smaller number of blades → Larger force per blade
→ smaller weight
→ reduced frictional losses due to fewer blade surfaces

Zweifel's Blade Loading Criterion

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Hence, there is an optimum value of blade spacing which will provide reasonably good efficiency as well as tangential momentum.

Zweifel's loading criterion is based on the non-dimensional force in a cascade.

$$Z = \frac{2F_{\theta}}{\rho V_2^2 C_z}$$

C_z axial chord

V_2 exit velocity

V_z axial velocity

F_{θ} blade force

Equating the work done to the enthalpy rise

$$F_{\theta}U = c_p(T_{o1} - T_{o2})\rho SV_z$$

From the relation

$$\psi = c_p \frac{(T_{o1} - T_{o2})}{U^2} = \frac{c_p \Delta T_o}{U^2} = \left[\frac{V_{z1}}{U} \tan \alpha_1 - \frac{V_{z2}}{U} \tan \alpha_2 \right]$$

and assuming that the flow is incompressible ($V_{z1} = V_{z2}$), it can be proven that

$$Z = \frac{2F_{\theta}}{\rho V_2^2 C_z} = 2 \cos^2 \alpha_2 \frac{S}{C_z} (\tan \alpha_1 + \tan \alpha_2)$$

α_2 is usually negative
for a turbine

Zweifel's Blade Loading Criterion

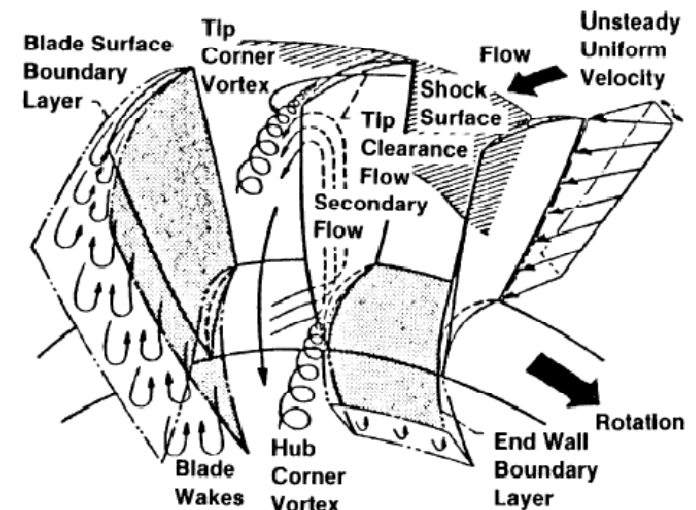
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- ❑ Zweifel suggested a value of 0.8 for the coefficient Z .
- ❑ The maximum turning allowed is dictated by the viscous effects and the Mach number considerations.
- ❑ Hence, a good performance can be obtained with
 - Very high turning and fewer number of blades
 - Small turning with a large number of blades
- ❑ Zweifel's criterion can be used to determine S/C_z or number of blades for a known value of α_1 and assumed value of α_2 .
- ❑ It has been used for compressible flows also with some success.

Losses in Turbine Blade Passages

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- a) **Profile loss** – associated with boundary layer growth over the blade profile (including separation loss under adverse conditions of extreme angles of incidence or high inlet Mach number).
- b) **Annulus loss** – associated with boundary layer growth on the inner and outer walls of the annulus.
- c) **Secondary flow loss** – arising from secondary flows which are always present when a wall boundary layer is turned through an angle by an adjacent curved surface.
- d) **Tip clearance loss** – near the rotor blade tip the gas does not follow the intended path, fails to contribute its quota of work output, and interacts with the outer wall boundary layer.



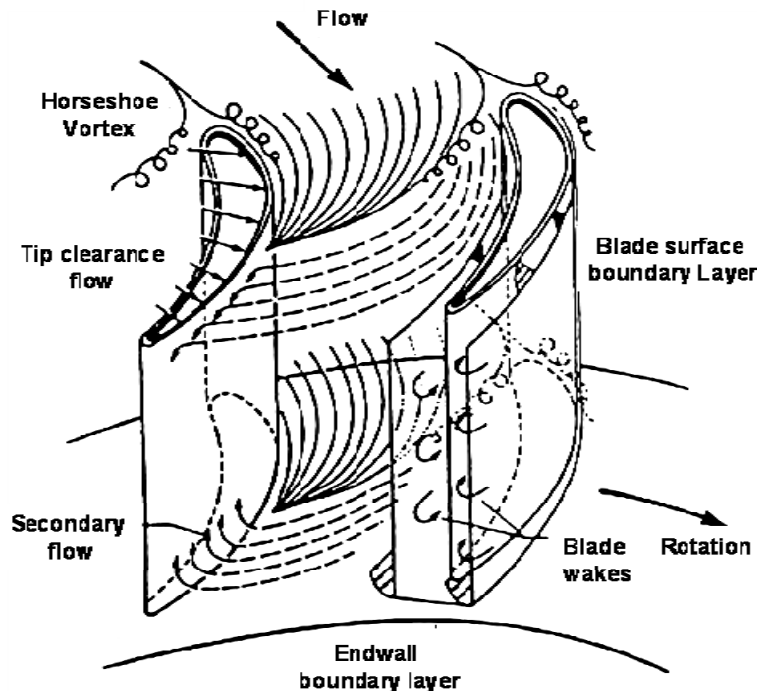
Viscous Effects

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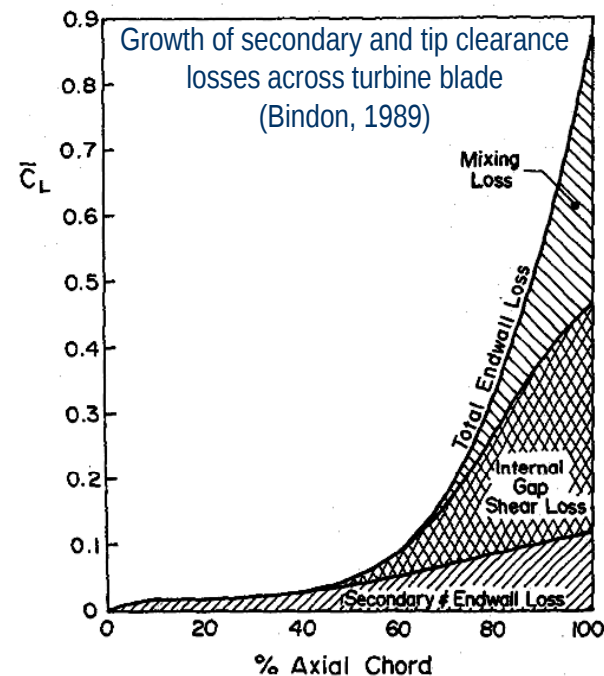
1. Decrease in turbine efficiency through stagnation pressure loss and increase in entropy
2. Decrease in pressure drop in turbines
3. Introduce three-dimensionality and change in flow properties, thus affecting the blade rows downstream also
4. Affect the cooling and heat transfer in turbine blades
5. Introduce flow blockage which affects mass flow rate and pressure drop
6. Introduce unsteadiness in the downstream blade rows owing to wake blade interaction; unsteady pressures generate blade vibration and noise

Complex Flow in Turbine Blade Passage PEMP RMD 2501

- ❑ Turbine passage flows are complex and three-dimensional.
- ❑ Major aerodynamic losses occurring in turbine blade passages are due to secondary and tip clearance flows (nearly 50-60% of total internal loss) and reduce efficiency by about 2-4%.



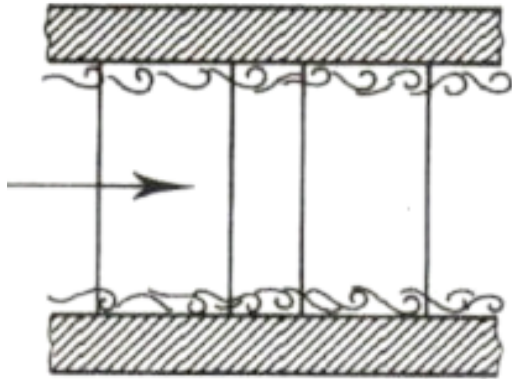
a) Turbine passage flow physics



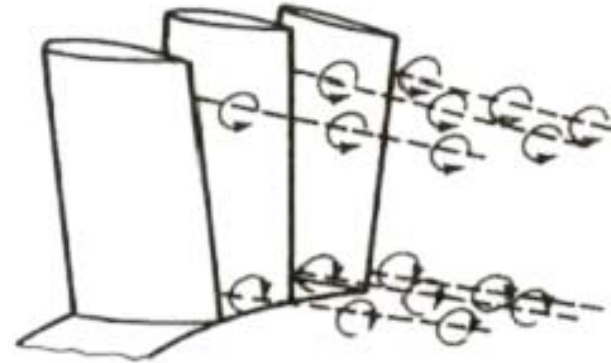
b) Internal turbine stage losses

Loss Mechanisms

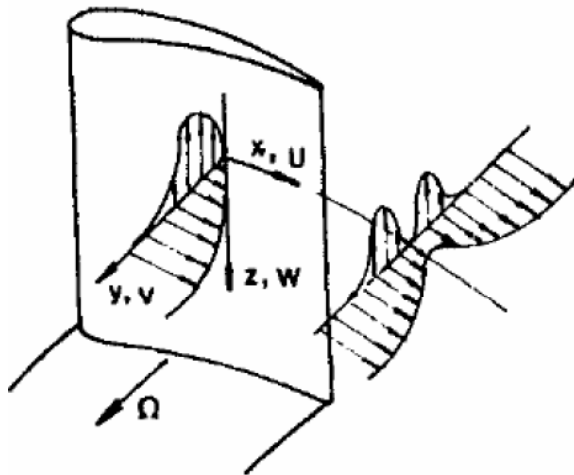
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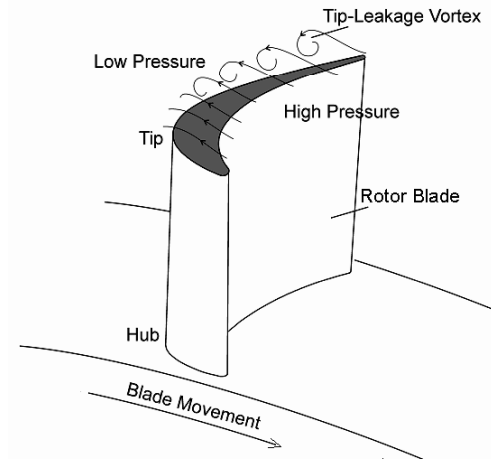
Annulus wall boundary layer loss



Secondary flow loss



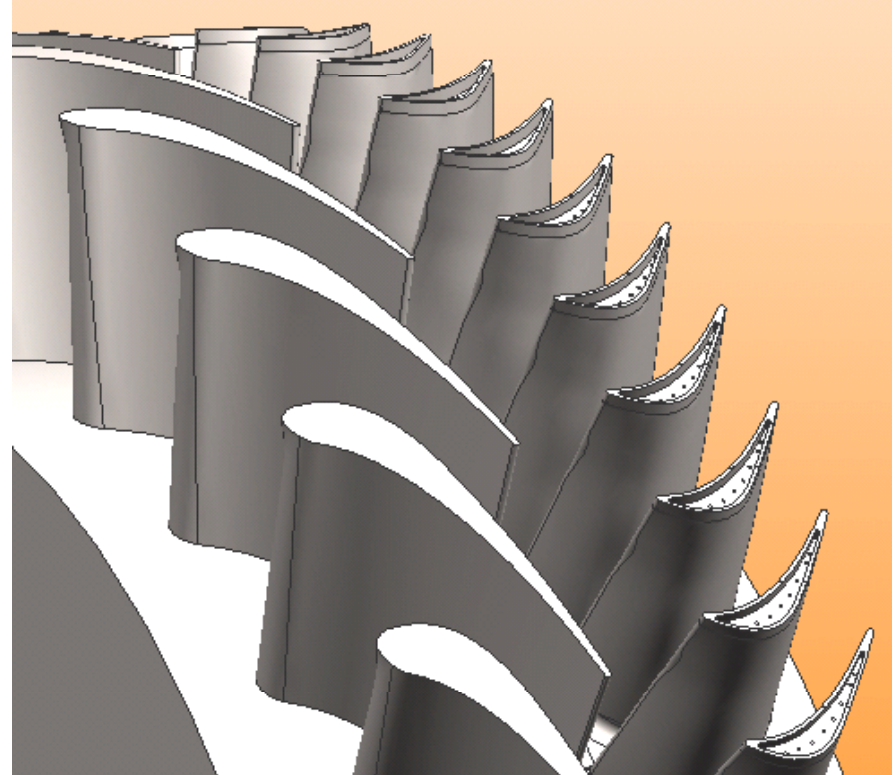
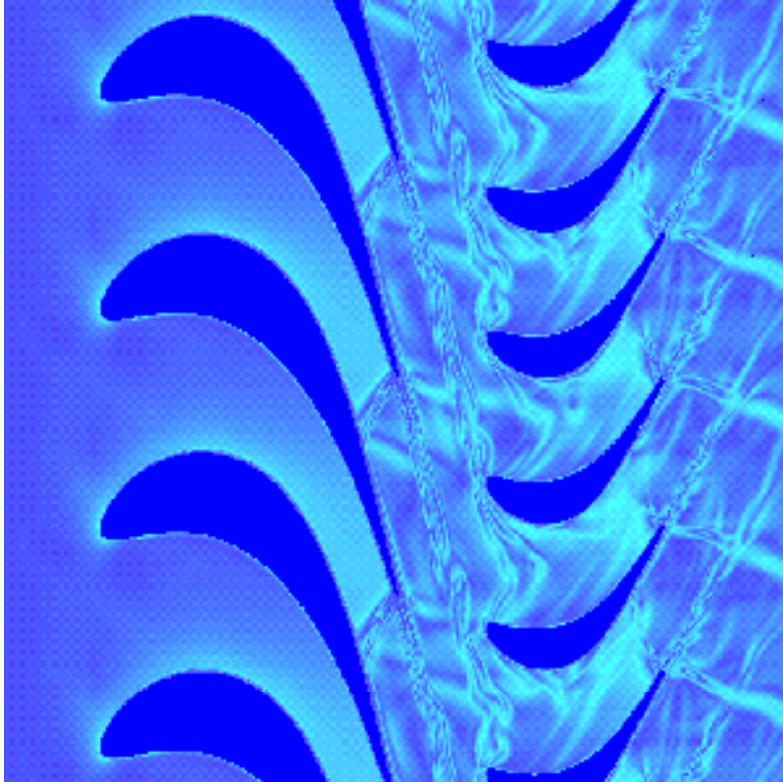
**Profile loss due to blade surface
boundary layer**



Tip clearance loss

Shock Waves in Transonic Turbines

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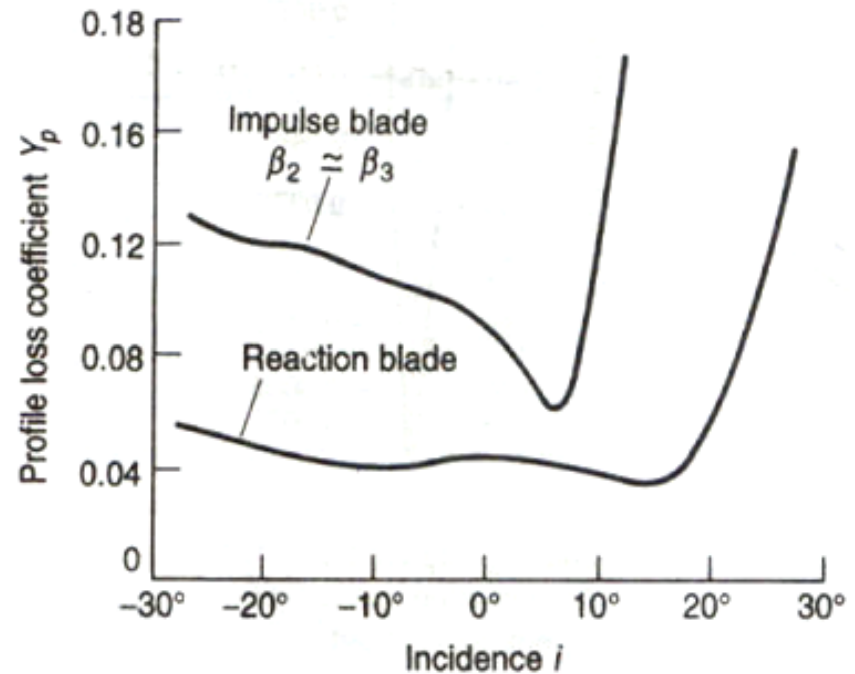


www-pgtu.de.unifi.it/CFDBranch/rd/eurd.html

Losses in Turbine Blade Passages

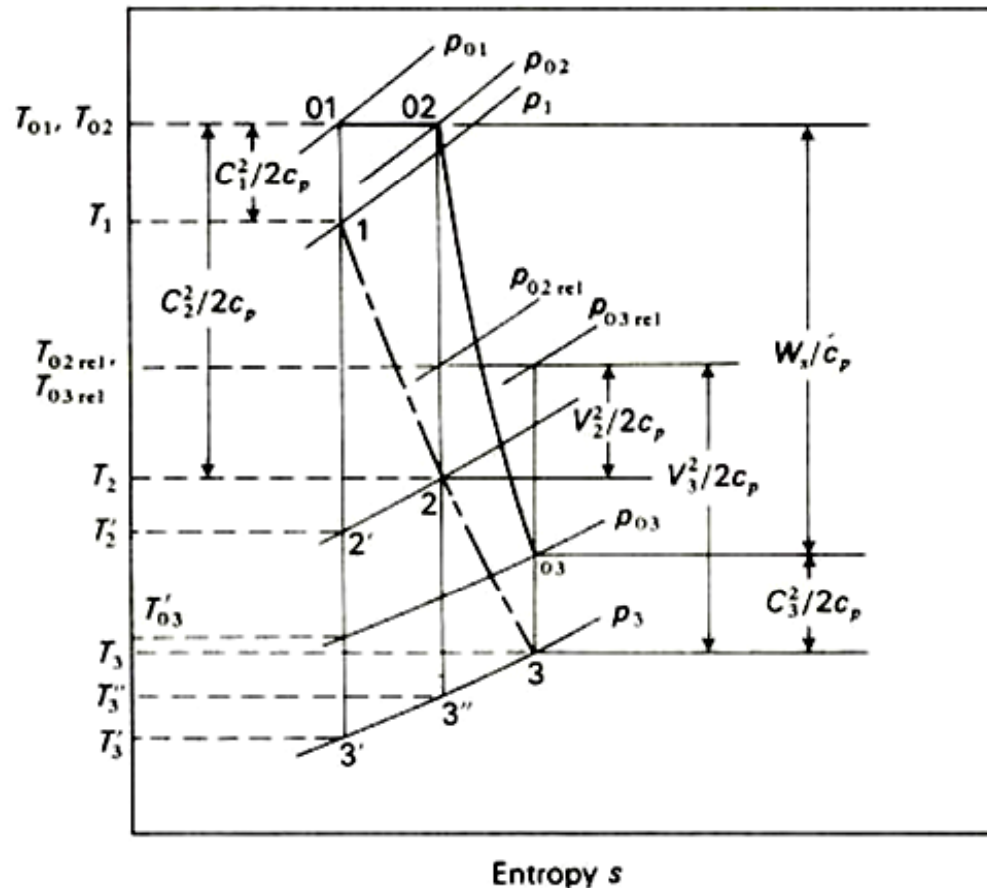
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- The profile loss coefficient Y_p is measured directly in cascade tests.
- Losses (b) and (c) cannot easily be separated, and they are accounted for by a secondary loss coefficient Y_s .
- The tip clearance loss coefficient, which normally arises only for rotor blades, will be denoted by Y_k .
- The total loss coefficient Y comprises the accurately measured two-dimensional loss, Y_p , plus the three-dimensional loss ($Y_s + Y_k$) which must be deduced from turbine stage test results.



Typical cascade results showing the effect of incidence on the profile loss coefficient Y_p for impulse ($R = 0$ and $\beta_2 \approx \beta_3$) and reaction type blading

Losses in Blade Rows



T-s diagram for a reaction stage

- ❑ Full line connects stagnation states.
- ❑ Chain dotted line connects static states.
- ❑ $T_{02} = T_{01}$ because no work is done in the nozzles; and the short horizontal portion of the full line represents the stagnation (pressure drop) $p_{01} - p_{02}$ due to friction in the nozzle.
- ❑ Ideally the gas in the nozzle would be expanded from T_{01} to T_2' but due to friction the temperature at the nozzle exit is T_2 , higher than T_2' .

Losses in Blade Rows

- Loss coefficient for the nozzle blade

$$\lambda_N = \frac{T_2 - T_2'}{C_2^2 / 2c_p} \quad \text{or} \quad Y_N = \frac{p_{01} - p_{02}}{p_{02} - p_2}$$

- Both λ and Y express the proportion of the leaving energy which is degraded by friction.
 - Y_N or λ_N is more easily used in design.
 - λ_N can easily be measured in cascade test.
- Isentropic expansion in the whole stage would result in a final temperature T_3' and in the rotor blade passage above T_3'' . Expansion with friction leads to a final temperature T_3 .

Losses in Blade Rows

- Similarly, loss coefficient for the rotor blade

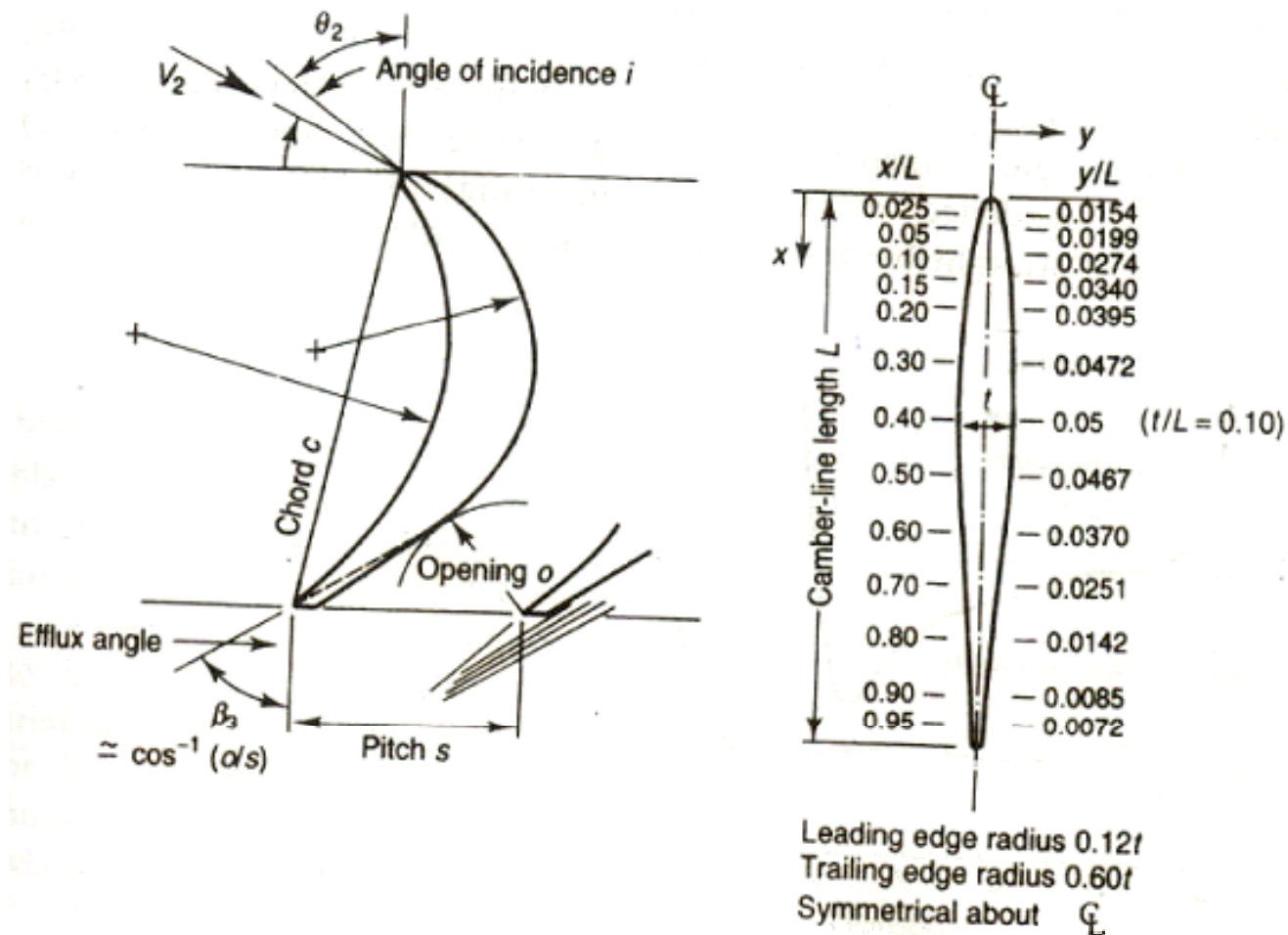
$$\lambda_R = \frac{T_3 - T_3''}{V_3^2 / 2c_p} \quad Y_R = \frac{p_{02rel} - p_{03rel}}{p_{03rel} - p_3}$$

- It can be shown that $Y_N \approx \lambda_N \left(\frac{T_{02}}{T_2'} \right)$ and $Y_R \approx \lambda_R \left(\frac{T_{03rel}}{T_3''} \right)$
- Also, λ_N and λ_R can be related to the stage isentropic efficiency

$$\eta_s \approx \frac{1}{1 + \frac{1}{2} \frac{C_a}{U} \left[\frac{\lambda_R \sec^2 \beta_3 + (T_3/T_2) \lambda_N \sec^2 \alpha_2}{\tan \beta_3 + \tan \alpha_2 - (U/C_a)} \right]} \quad (1)$$

Cascade Nomenclature

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Estimation of Stage Performance

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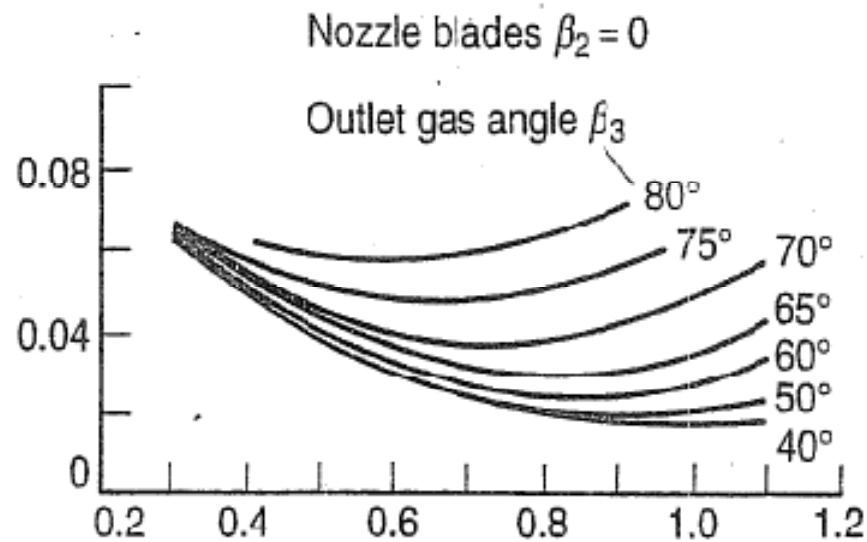
- Estimate $(Y_p)_N$ and $(Y_p)_R$ from the gas angles of the proposed design by using the loss data in conjunction with the interpolation formula,

$$Y_p = \left\{ Y_{p(\beta_2=0)} + \left(\frac{\beta_2}{\beta_3} \right)^2 \left[Y_{p(\beta_2=\beta_3)} - Y_{p(\beta_2=0)} \right] \right\} \left(\frac{t/c}{0.2} \right)^{\beta_2/\beta_3} \quad (2)$$

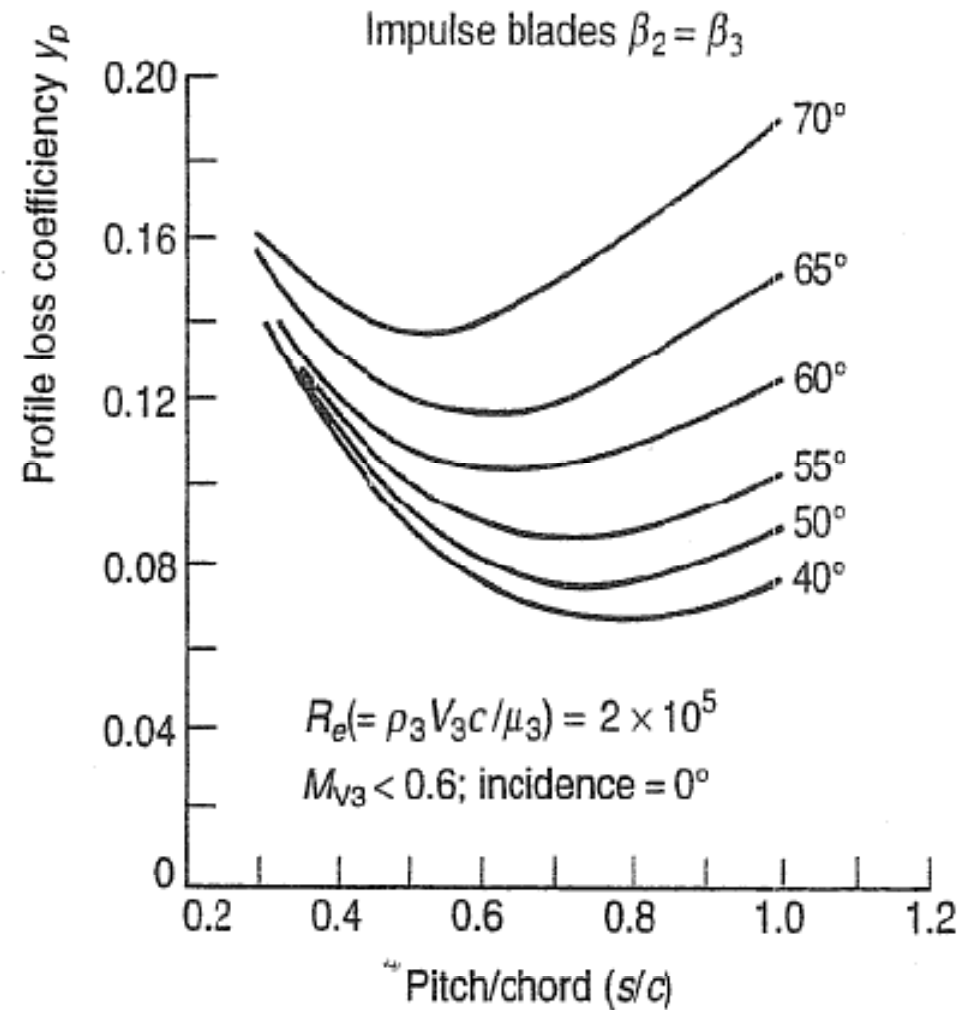
- This equation represents a correction for a change in inlet angle at a constant outlet angle, so that $Y_{p(\beta_2=0)}$ and $Y_{p(\beta_2=\beta_3)}$ are the values for a nozzle and impulse-type blade having the same outlet gas angle β_3 as the actual blade. Equation also includes a correction for t/c if it differs from 0.2, a reduction in t/c leading to reduced profile loss for all blades other than nozzle-type blades ($\beta_2 = 0$). The degree of acceleration of the flow in the blading decreases with the degree of reaction as $\beta_2/\beta_3 \rightarrow 1$, and the influence of blade thickness becomes more marked as the acceleration is diminished. The correction is considered reliable only for $0.15 < t/c < 0.25$.

Profile Loss Coefficient

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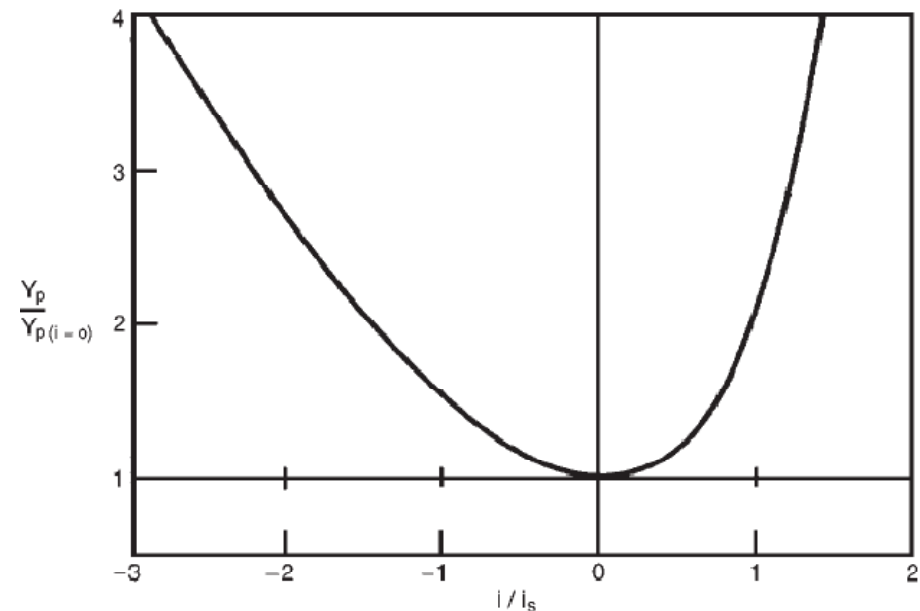


Profile loss coefficient for conventional blading with $t/c = 0.20$



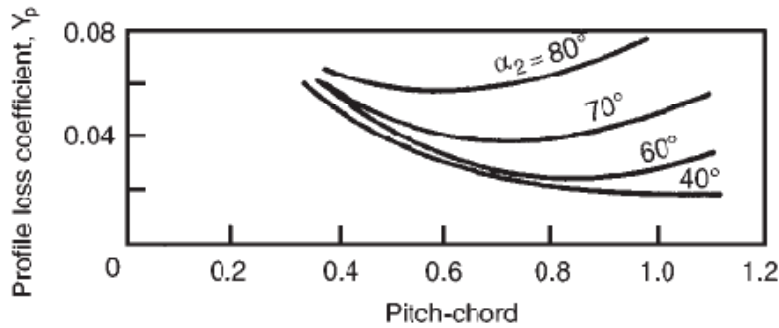
Correction for Incidence

- ❑ If the blades operate at a non-zero incidence at the design point, a correction to Y_p would be required.
- ❑ This correction is really only important when estimating performance at part load.
- ❑ It involves using correlations of cascade data to find the stalling incidence i_s for the given blade (i.e. incidence at which Y_p is twice the loss for $i = 0$); and then using a curve of $Y_p/Y_p(i=0)$ versus i/i_s to find Y_p for the given i and the value of $Y_p(i=0)$.

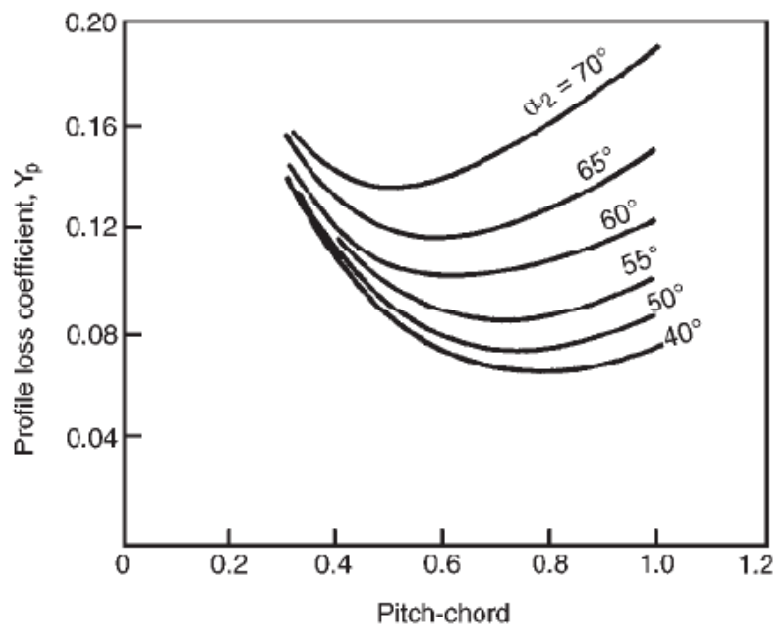


Variation of profile loss with incidence for typical turbine blading (adapted from Ainley and Mathieson 1951)

Loss Correlations – Profile Loss



(a) Nozzle blades, $\alpha_1 = 0$



(b) Impulse blades, $\alpha_1 = \alpha_2$

($t/l = 0.2$ $Re = 2 \times 10^5$; $M < 0.6$)

Ainley and Mathieson correlated the profile losses of turbine blade rows against space/chord ratio s/l , fluid outlet angle α_2 , blade maximum thickness/chord ratio t/l and blade inlet angle. The variation of $Y_p = Y_{p(i=0)}$ against s/l is shown here for nozzles and impulse blading at various flow outlet angles. For other types of blading intermediate between nozzle blades and impulse blades the following expression is employed:

$$Y_{p(i=0)} = \left\{ Y_{p(\alpha_1=0)} + \left(\frac{\alpha_1}{\alpha_2} \right)^2 [Y_{p(\alpha_1=\alpha_2)} - Y_{p(\alpha_1=0)}] \right\} \left(\frac{t/l}{0.2} \right)^{\alpha_1/\alpha_2}$$

where all the Y_p 's are taken at the same space/chord ratio and flow outlet angle.

Profile loss coefficients of turbine nozzle and impulse blades at zero incidence ($t/l = 20\%$; $Re = 2 \times 10^5$; $M < 0.6$) (adapted from Ainley and Mathieson 1951).

Loss Correlations – Secondary Flow Loss^{PEMP RMD 2501}

Secondary Flow Loss

The secondary losses arise from complex three-dimensional flows set up as a result of the end wall boundary layers passing through the cascade.

1. Ainley's Correlation: $C_{Ds} = \lambda C_L^2 / (s/l)$

where λ is a parameter, which is a function of the flow acceleration through the blade row. For incompressible flow, $C_D = Y(s/l) \cos^3 \alpha_m / \cos^2 \alpha_2$

hence
$$Y_s = \frac{C_{Ds} \cos^2 \alpha_2}{(s/l) \cos^3 \alpha_m} = \lambda \left(\frac{C_L}{s/l} \right)^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} = \lambda Z$$

where Z is the aerodynamic loading coefficient.

2. Dunham and Came Correlation:

$$Y_s = 0.0334 \left(\frac{l}{H} \right) \left(\frac{\cos \alpha_2}{\cos \alpha_1'} \right) Z$$

This represents significant improvement over Ainley's correlation

Loss Correlations – Secondary Flow Loss^{PEMP RMD 2501}

Secondary Flow Loss

Recently, more advanced methods of predicting losses in turbine blade rows have been suggested which take into account the *thickness* of the entering boundary layers on the annulus walls.

3. Came's Correlation:

$$Y_s = \left(0.25Y_1 \frac{\cos^2 \alpha_1}{\cos^2 \alpha_2} + 0.009 \frac{l}{H} \right) \left(\frac{\cos \alpha_2}{\cos \alpha_1'} \right) Z - Y_1$$

This is a modified form of Dunham's correlation, and represents the net secondary loss coefficient for *one* end wall only and where Y_1 is a mass-averaged inlet boundary layer total pressure loss coefficient. It is evident that the increased accuracy obtained by using this relation requires the additional effort of calculating the wall boundary layer development.

Secondary and Tip Clearance Losses PEMP RMD 2501

- Secondary and tip-clearance loss data for Y_s and Y_k have been correlated using the concepts of lift and drag coefficient. (*ref. notes on axial compressor*).

$$C_L = 2(s/c)(\tan \beta_2 + \tan \beta_3) \cos \beta_m$$

where

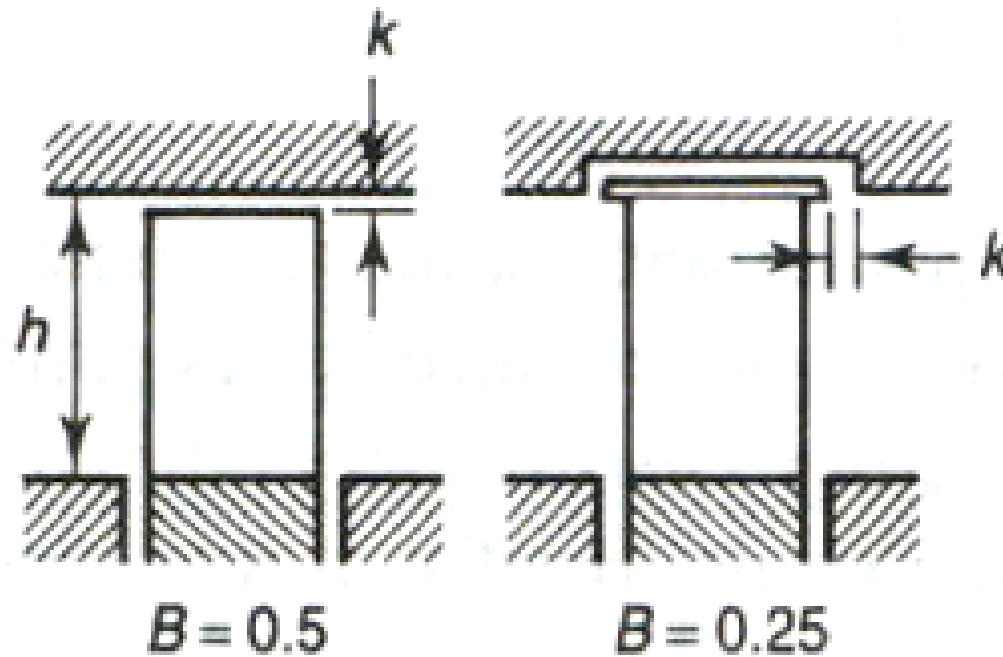
$$\beta_m = \tan^{-1}[(\tan \beta_3 - \tan \beta_2)/2]$$

- It is convenient to treat Y_s and Y_k simultaneously. The proposed correlation is

$$Y_s + Y_k = \left[\lambda + B \left(\frac{k}{h} \right) \right] \left[\frac{C_L}{s/c} \right]^2 \left[\frac{\cos^2 \beta_3}{\cos^3 \beta_m} \right] \quad (3)$$

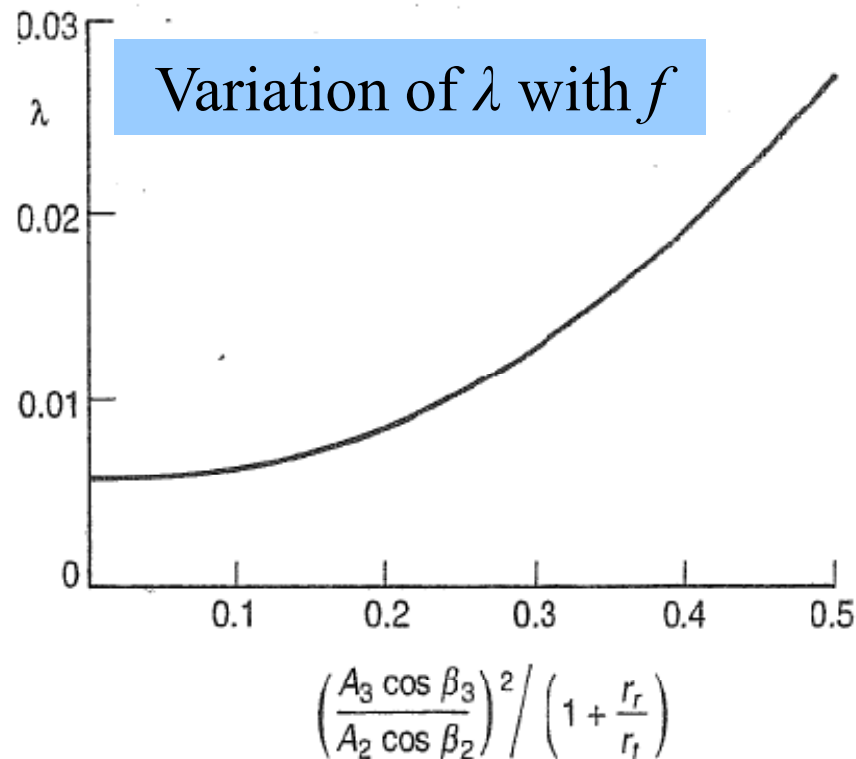
Definition of B , h and k

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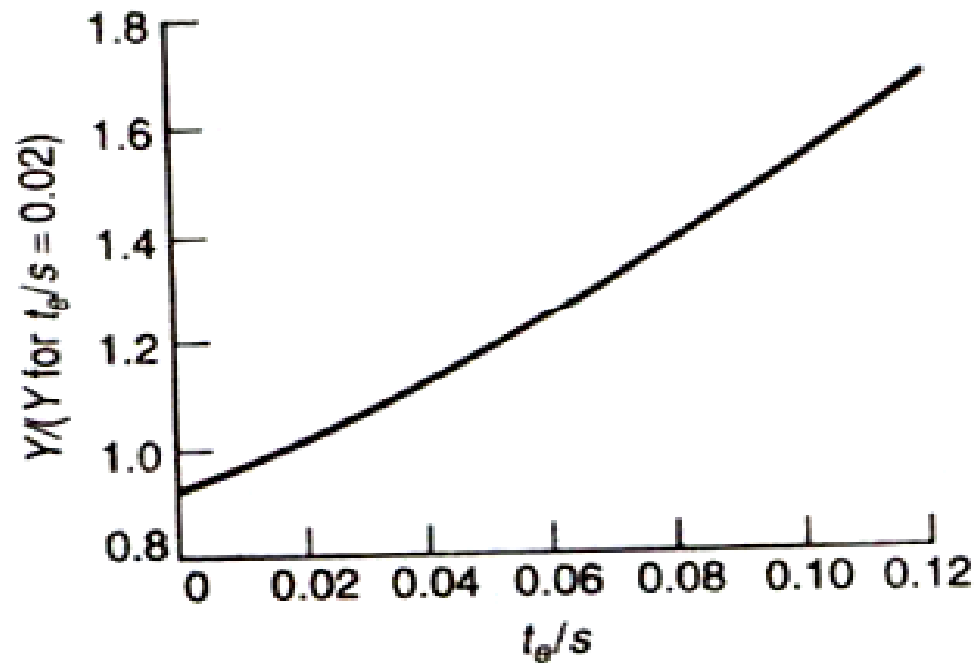
Definition of λ and f

- The degree of acceleration is indicated by the ratio of the areas $A_3 \cos \beta_3 / A_2 \cos \beta_2$ normal to the flow direction.
- The quantity λ is approximately given by:
$$\lambda = f \left\{ \left(\frac{A_3 \cos \beta_3}{A_2 \cos \beta_2} \right)^2 / \left(1 + \frac{r_r}{r_1} \right) \right\}$$



Correction for Trailing Edge Thickness

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Correction to be applied if the trailing edge thickness/pitch ratio (t_e/s) differs from 0.02

Correction for M and Re

- Cascade data and other loss correlations are strictly applicable only to designs where **Mach numbers** are such that no shock losses are incurred in the blade passages.
- If the outlet relative Mach number is greater than unity, then the following correction should be applied to Y_p obtained from equation (2):

$$Y_{p\,corr} = \left[Y_p \text{ from eqn(2)} \right] \times \left[1 + 60(M - 1)^2 \right]$$

(M is exit relative Mach number for rotor blades and exit absolute Mach number for nozzles)

- If the **mean Reynolds number** of the turbine based on blade chord differs much from 2×10^5 , then an approximate correction to the overall isentropic efficiency is given by:

$$(1 - \eta_t) = \left(\frac{Re}{2 \times 10^5} \right)^{-0.2} (1 - \eta_t)_{Re=2 \times 10^5}$$

Correction for λ and B

- The Ainley-Mathieson correlations outlined here predict efficiencies to within $\pm 3\%$ of measured values.
- The method becomes applicable to a wide range of turbines, including small gas turbines, if the following corrections suggested by Dunham and Came are incorporated.
- Calculate λ by the expression

$$\lambda = 0.0334 \left(\frac{c}{h} \right) \frac{\cos \beta_3}{\cos \beta_2}$$

$\beta_3 = \alpha_2$ and $\beta_2 = \alpha_1$
for nozzles

- Replace $B(k/h)$ by the expression

$$B \left(\frac{c}{h} \right) \left(\frac{k}{c} \right)^{0.78}$$

**$B = 0.47$ for radial tip clearance
 $= 0.37$ for side clearance on
shrouded blades**

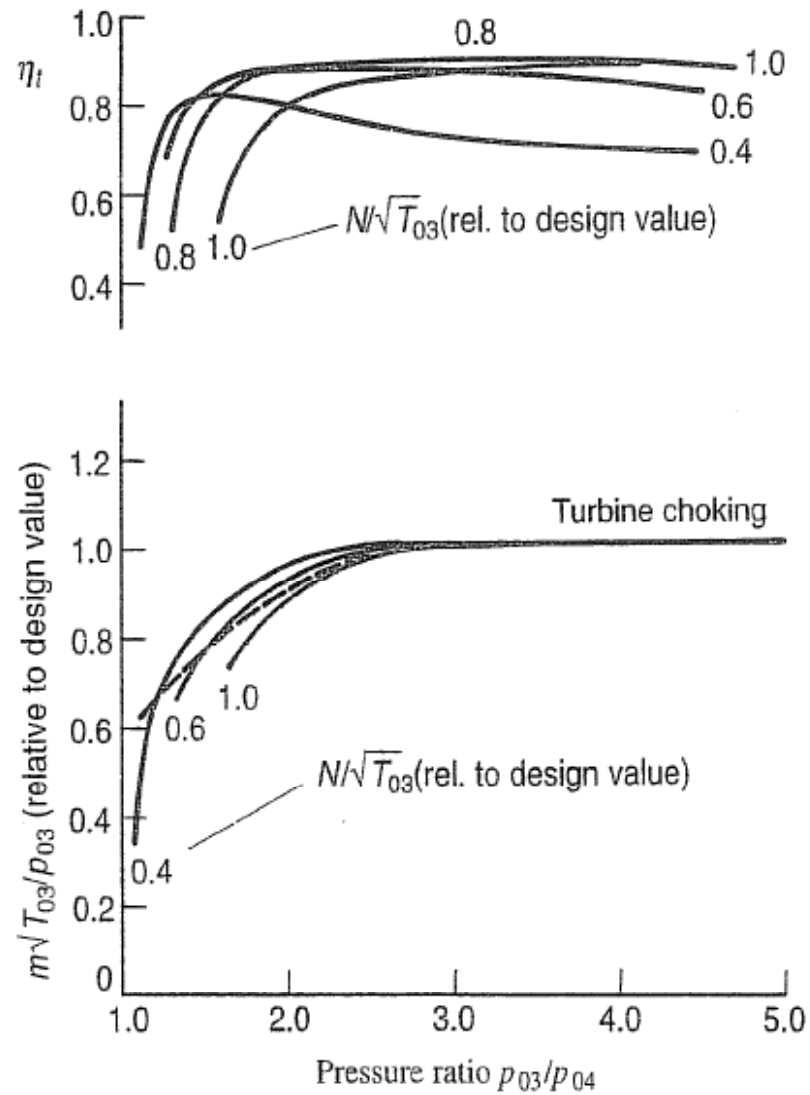
Estimation of Turbine Performance

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1. Estimate $(Y_p)_N$ and $(Y_p)_R$ from eqn.(2) applying corrections for incidence, Mach number and trailing thickness
2. Estimate $(Y_s + Y_k)$ from eqn.(3) applying corrections for the parameters B and λ
3. Calculate total losses Y_N and Y_R
4. Calculate $\lambda_N = \frac{Y_N}{(T_{02}/T_2')}$ and $\lambda_R = \frac{Y_R}{(T_{03rel}/T_3'')}$
5. Calculate **turbine stage efficiency** from eqn.(1)
6. Apply Re correction if required
7. Repeat the calculations for off-design conditions

Turbine Performance

PEMP
RMD 2501



Session Summary

PEMP
RMD 2501

- Basic construction and application of axial turbines is discussed.
- Aerothermodynamics of turbines is explained through T-s diagram and velocity triangles.
- Selection of whirl distribution is discussed based on the radial equilibrium theory.
- Loss mechanism in turbine blade rows and various loss correlations are discussed.
- Turbine blade loading criteria and performance characteristics are explained in detail.

Thank you